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Environmental risk and welfare valuation under imperfect information

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Abstract

Consumers are often uninformed, or unsure, about the ambient level of environmental risk. An optimal policy must jointly determine efficient levels of self-protection, information provision, and public risk mitigation efforts. Unfortunately, conventional welfare measures are not amenable to welfare analysis in the presence of imperfect information. We develop a theoretical welfare measure, called quasi-compensating variation, that is a natural extension of compensating variation (CV). We show that this welfare measure offers not only a money metric of the ''value of information,'' but also a means to appropriately evaluate the welfare effects of various policies when consumers are imperfectly informed about ambient risk. This welfare measure allows us to obtain a number of results that the traditional CV measure fails to offer. In particular, we show that the consumer's willingness to pay for a (small) environmental risk reduction is higher for those who underestimate ambient risk than for those who overestimate or are perfectly informed if the marginal return to self-protection increases with ambient risk.
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1. Introduction

The most pernicious effects of environmental damage are often experienced through impaired human health, and determining the optimal level of public pollution abatement is an important part

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of environmental economics.[1] Conventional wisdom suggests that the optimal policy must equate the aggregate marginal cost with the aggregate marginal benefit of public abatement. Over the years, environmental economists have tried to elicit the welfare value of environmental risk reductions through non-market valuation based on stated preferences or revealed preferences or both. An important complication arises in this endeavor when consumers can and do take costly steps to avoid adverse health impacts.

In early theoretical studies, non-stochastic models such as those of Courant and Porter (1981) or Bartik (1988) showed that the marginal benefit of exogenous pollution reduction equals the cost of self-protection, translated via the marginal rate of technical substitution between pollution reduction and self-protection. Berger et al. (1987), in a two-outcome model, extended the result to the case of stochastic health outcomes. Bartik (1988) also showed that, for non-marginal changes, expenditures on self-protection serve as a lower-bound estimate for willingness to pay (WTP) for pollution reduction. In a continuous-state stochastic model, however, Shogren and Crocker (1991, p. 13) argued that "unobservable utility terms cannot be eliminated from marginal willingness-to-pay expressions, implying that empirical efforts which identify marginal rates of substitution with willingness-to-pay are misdirected." This is the earliest notable criticism we have found against the conventional argument that the efficient level of pollution risk reduction is that which equates the marginal cost of private self-protection with that of public pollution abatement. Using a state-contingent formulation, Quiggin (1992) derived a more positive result that the conventional relationship can be obtained, but only under certain assumptions. Shogren and Crocker (1999) later criticized the restrictiveness of Quiggin's separability assumption, arguing again that without it, averting expenditures are an unreliable measure of willingness to pay.

We add a new dimension to this important discussion. In both Shogren and Crocker (1991) and Quiggin (1992), individuals observe the true level of exogenous ambient hazard (or, equivalently, "ambient risk") and choose a level of self-protection, but a random element causes uncertain health outcomes.[2] Given the distribution of stochastic health outcomes, individuals in the model optimize as well as anyone could. They are perfectly informed, and thus form perfectly accurate perceptions about health risks. In this sense, their self-protection levels are *ex ante* optimal. However, a number of empirical studies have indicated that consumers are often imperfectly informed, and have incorrect perceptions, about ambient risk level itself.[3] Misperceptions about ambient environmental risk imply, among other things, that expenditures on self-protection can sometimes be far from optimal and are therefore an unreliable estimate of willingness to pay.

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[1] Two recent issues of *Environmental and Resource Economics* were devoted to studies on the valuation of health risks and its implications for environmental policies (vol. 33, no. 3 and vol. 34, no. 3).

[2] There can be other types of randomness such as in the effectiveness of self-protection and preference ordering. See, for example, Quiggin (2002).

[3] Abdalla et al. (1992) reported that in a survey of households in Pennsylvania, only 43.2% of survey respondents were aware of the trichloroethylen (TCE) contamination. This is despite the government's mandatory notification of the contamination problem. Powell (1991) and Walker et al. (2006) also reported that the survey respondents' beliefs about their drinking water quality were considerably heterogeneous. See also U.S. GAO (1992), Collins and Steinback (1993), and Abrahams et al. (2000). The heterogeneity in consumers' information structures and perceptions comes partly due to an important toxicological property that is inherent in many environmental pollutants: Humans may experience cancer or other chronic health effects only after a long period of exposure to certain levels whereas every dose or exposure is associated with some increased health risk. Because health outcomes are stochastic and often experienced only after a sufficient passage of time, some consumers fail to recognize direct causal relationships. One may wish to define welfare measures incorporating these dynamics. As in previous theoretical models, we do not explicitly model such dynamics in our welfare measure. Nonetheless, we must acknowledge that such dynamics are implicit in all valuation studies that deal with environmental risks.

To illustrate the way in which averting expenditures can lead one far astray, consider a toxic water contaminant, which causes no health effect and a serious, chronic health condition, respectively, at a low and a high concentration level. A personal treatment device can achieve the low concentration level. Suppose that concentrations in a local water supply are currently high. An informed consumer, understanding the concentration level and its threat, would purchase the device and so protect herself. If a regulator were to reduce concentrations to the low level, the benefit to this person is simply the cost of the device, which would become unnecessary. An uninformed consumer, thinking that the water is clean, would not protect. Absent intervention by the government, this person is doomed to suffer the health consequences of the toxin. The benefit of the collective cleanup to her is very large. Yet if cleanup does not occur, observed averting expenditures for the first person are the cost of the device (the correct estimate), and for the second person zero (a potentially vast underestimate).

Our goal is to understand how this kind of uncertainty, in consumers' perceptions about ambient risk levels, affects self-protection expenditures, welfare valuation, and environmental policy evaluation in general. Our specification of uncertainty allows us to address this question. We depart from previous literature in our specification of uncertainty for another reason as well: we wish to explore the trade-off that the regulator might often need to make between spending money on cleanup and spending money to inform and educate the public so that they will protect themselves. It is certainly possible that in some cases the least expensive avenue leading to overall optimality is for individuals to protect themselves. In a world where consumers are perfectly informed, the marginal cost of ''public'' abatement should equal the marginal cost of ''private'' abatement (*i.e.* self-protection), at the optimum. However, this need not be the case in reality. One needs to determine efficient levels of self-protection, public abatement, and information provision jointly. A difficulty in coming at this problem is the apparent lack of a necessary welfare-measurement apparatus that enables us to evaluate the benefits of public abatement and information provision for imperfectly informed consumers.

To formalize these ideas, we develop a model in which individuals are uncertain about the ambient risk level itself. Each consumer possesses a subjective belief about ambient hazard and makes a possibly suboptimal self-protection choice, which then leads to possibly non-optimal health outcomes. With this setup, we propose a new welfare measure, called quasi-compensating variation (QCV). Our measure gives meaningful results in the presence of imperfect information in that it offers a means to value the welfare effects of various policies when consumers are imperfectly informed.[4] It also coincides precisely with conventional compensating variation when consumers are perfectly informed. We also show how use of this measure, rather than the usual CV measure, can yield different, more sensible policy implications. In particular, we show that the consumer's WTP for a small change in ambient environmental risk is higher for those who underestimate ambient risk than for those who overestimate or are perfectly informed if the marginal return to self-protection increases with ambient risk. In short, when consumers are wrong about the exogenous ambient hazard, our welfare measure produces the correct measure of willingness to pay and therefore permits us to perform policy evaluation correctly.

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[4] There is large literature on the value of information. Our welfare measure is very similar to the way many authors define the value of information when only information changes. To our knowledge, however, ours is the first attempt to define a money metric of the welfare value of a change in both informational and non-informational parameters under imperfect information.

The layout of the paper is as follows. In Section 2, we describe the basic setup of the model, clarify terminology and provide a formal definition of QCV. In Section 3, we provide several comparative statics results of interest to policy analysis. Our analyses are primarily confined to the case in which the only source of uncertainty is in the subjective beliefs regarding exogenous ambient hazard outlined above. In Section 4, we present briefly a generalization of the model to the case in which the health production function is also stochastic. In Section 5, we discuss how our welfare measure relates to contingent valuation. The last section concludes.

2. The model

2.1. The model with perfect information

We first set up our model with perfect information and derive its essential properties. Consider a composite ambient risk $r \in [0, r_{\max}]$ that poses environmental risks to humans. In the case of water pollution, for example, it may be a composite of l possible toxic chemicals. For simplicity, we assume that a linear combination of n defensive measures gives rise to a "smooth" composite self-protective measure s. For the moment, we assume that the health production function $h(s, r)$ relating the level of ambient risk and the level of self-protection is deterministic. Thus, consumers do not face uncertainty with regard to health outcomes given their choice of self-protection and ambient risk. We also ignore the joint production of health and utility. Finally, we assume that individual preferences are given by $u = u(z, h)$, where z is a numeraire, and that individuals are risk-averse.

Some regularity conditions are imposed on the utility and health production functions:

A1. u is strictly quasi-concave in (z, h) and twice differentiable with $u_z > 0, u_{zz} < 0, u_h > 0, u_{hh} < 0$.

A2. h is strictly concave in s for each fixed r and twice differentiable with $h_r < 0, h_s > 0, h_{ss} < 0$.

All assumptions are standard and are used explicitly or implicitly in the theoretical papers cited above. Note that we did not specify the sign of h_{sr}, which may be positive or negative depending on the characteristics of ambient risks. Courant and Porter (1981), and Bartik (1988) assume that $h_{sr} < 0$ whereas Quiggin (1992, R.2, p. 47) assumes that $h_{sr} \geq 0$, which corresponds to "strong nonconvexity" in Shogren and Crocker (1991). The assumption $h_{sr} \geq 0$ means that the marginal return to self-protection increases with the level of ambient risk. That is, self-protection is more effective when the contaminant level is higher. For many environmental risks of interest, such as radon in air or arsenic in water, $h_{sr} \geq 0$ seems to be reasonable. As will be shown below, the assumption regarding the sign of h_{sr} turns out to be important in signing the direction of welfare impacts.

A series of preliminary results are in order, which support the primary results to come. The consumer's utility-maximization problem is:

$$\max_{z,s} \ u(z, h(s, r)), \quad \text{s.t.} \ z + ps \leq m, \tag{1}$$

where p is the price of self-protection and m is the income. Let $z^*(p, m, r)$ and $s^*(p, m, r)$ denote solutions to (1) and symbol '$*$' represent an optimal solution throughout the paper. The

"price" of health in the (z, h)-space equals the marginal rate of substitution, evaluated at the optimal s^*:

$$P(r) = \frac{p}{h_s(s^*(p, m, r), r)}. \tag{2}$$

From expression (2), it is obvious that $P(r)$ depends endogenously on the consumer's choice s^* unless h exhibits constant return to scale. Due to this endogeneity, "Marshallian demand functions for [household] commodities [*i.e.* h] cannot be uniquely determined ... nor can compensating and equivalent variation measures be bounded by Marshallian consumer's surplus" (Bockstael and McConnell, 1983, p. 806). In fact, in our specific context, we can make an even stronger statement. That is, compensating (equivalent) variation and compensating (equivalent) surplus are not well-defined as a money metric in (z, h)-space. To overcome this difficulty, Bockstael and McConnell showed that the area under the Hicksian demand for goods (*i.e.* s) can serve as a welfare measure for changes in r. Our approach is slightly different from Bockstael and McConnell, in that we define CV and EV measures in (z, s)-space by showing that a well-defined preference order in the (z, h)-space can be transformed to that in the (z, s)-space.

Lemma 1. *Under* A1 *and* A2, *the following hold.*

  (i) *Define a transformed utility function in (z, s)-space, $u(z, s; r) = u(z, h(s, r))$. u is strictly quasi-concave in (z, s) for each fixed r.*
 (ii) *Let $r_0 \neq r_1$ and let $\bar{v} \in \mathbb{R}$ be fixed. Then, two indifference curves $I(\bar{v}, r_0) = \{(z, s) : u(z, s; r_0) = \bar{v}\}$ and $I(\bar{v}, r_1) = \{(z, s) : u(z, s; r_1) = \bar{v}\}$ cannot cross.*
(iii) *The optimal choice of self-protection s^* is continuous in r.*
(iv) *Suppose that $h_{sr} \geq 0$ and that either u is homothetic in (z, h) or $u_{zh} = 0$. Then the optimal choice of alleviating options s^* is non-decreasing in r.*

Proof. See Appendix A. □

Lemma 1(i and ii) state that, under reasonable assumptions, the transformed indifference curves in (z, s)-space are well-behaved. For each fixed utility level, the corresponding level curve will be shifted inwards (*i.e.* to the southwest in (z, s)-space) as the ambient risk decreases. Thus, a reduction in ambient risk shifts the entire family of indifference curves defined in the (z, s)-space. This result is appealing intuitively. As the ambient level of risk decreases, the need for self-protection decreases, and therefore the marginal rate of substitution between goods z and s changes. Lemma 1(iv) combined with Lemma 1(i and ii) implies that the optimal vector (z^*, s^*) moves upwards along the budget line *in the (z, s) - space* as the ambient risk decreases.

2.2. The model with imperfect information

To formalize the idea of imperfect information, we introduce the consumer's *subjective* belief about the distribution of the ambient risk r. Let $\mathcal{F}$ be the set of cumulative distribution functions whose support is confined to $[0, r_{\max}]$. For each individual i, $F_i \in \mathcal{F}$ describes i's belief about the ambient risk. As a special case, this formulation allows for (i) a degenerate distribution F such that $Pr_F\{r = r^c\} = 1$ (*i.e.* an individual believes with certainty that the ambient risk is some value r^c) and (ii) a belief F such that $\hat{r} \notin \text{Supp}(F)$ where $\hat{r}$ is the "true" ambient risk (*i.e.* an

individual's belief is completely off the true ambient risk). We assume throughout that there is an objective measure of the true ambient risk $\hat{r}$. Though it may be difficult to specify a truly objective measure, this assumption is consistent with the existing literature (*e.g.* Shogren and Crocker, 1991; Quiggin, 1992; Bresnahan and Dickie, 1995).[5]

Given a belief F, an individual solves

$$\max_{z,s} \int u(z, h(s, r)) \, \mathrm{d}F(r), \quad \text{s.t. } z + ps \leq m. \tag{3}$$

The major task of this paper is to evaluate welfare changes due to changes in F, p, or $\hat{r}$.[6]

The implicit assumption underlying CV and EV measures is that consumers whose welfare impacts are in question are able to make their consumption choices optimally. Previous empirical analyses have estimated either perfectly informed consumers' WTP or imperfectly informed consumers' WTP or both, but have not explicitly discussed how one might define the welfare values for imperfectly informed consumers (see Kim and Cho, 2002; Chien et al., 2005 for contingent-valuation studies, Murdock and Thayer, 1990; Abdalla et al., 1992; Collins and Steinback, 1993; Laughland et al., 1993; Abrahams et al., 2000 for averting-expenditure studies, and Harrison and Rubinfeld, 1978; Portney, 1981; Murdock and Thayer, 1988; Gayer et al., 2000; Kim et al., 2003 for hedonic studies). While some of these studies do acknowledge the effects of consumers' perceptions or information structures and correct for the (presumably systematic) bias, none of them incorporates imperfect information in the same manner as we do.

In both Shogren and Crocker (1991) and Quiggin (1992), consumers observe the true level of exogenous ambient risk and choose a level of self-protection, but a random element causes uncertain health outcomes. The consumers are perfectly informed about the distribution of stochastic health outcomes. Thus, the consumers in their models optimize as well as anyone could, and their self-protection levels are *ex ante* optimal. In contrast, consumers in our model are imperfectly informed, and thus uncertain, about the degree of ambient risk. Thus, the consumers may choose a suboptimal level of s based on their subjective belief F. Because health outcomes are endogenous in s, the consumers face a suboptimal level of health risks for a given $\hat{r}$. Changes in $\hat{r}$, therefore, affect the welfare levels of informed consumers differently than those of uninformed consumers. The existing literature appears silent on how to define willingness to pay for exogenous risk-reduction for imperfectly informed consumers when the welfare effects of such risk-reduction are endogenous in the choice variable.[7] Ideally, we would like to have a

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[5] It is well known that health risks often depend on individual-specific factors such as genes, age, sex and co-morbidities. In our formulation, these are reflected in the health production h and not in $\hat{r}$. This enables us to speak unambiguously of "exogenous changes in $\hat{r}$."

[6] One point must be clarified. Throughout the paper, by "information," we refer to information concerning the ambient risk level, but not information about the probability distribution of possible health outcomes. Therefore, F captures neither the consumer's risk beliefs nor risk attitudes. In other words, the consumer (and the regulator) knows with certainty what health effects she would have if she knows the true ambient risk, say, the quality of water. As evidenced by Dickie and Gerking (1996), the consumer's perception about the chance of contracting a disease (*i.e.* risk belief) may be one of the key determinants of the willingness to pay to avoid the disease. This risk perception may be highly correlated with the degree of imperfect information. This point is further discussed in Section 4.

[7] There is a literature investigating the complex influence of consumers' incomplete beliefs on non-market valuations (see Crocker et al., 1998). While they focus on the institution-dependence of *expressed* welfare values due to consumers' incomplete or incoherent beliefs, we focus on the dependence of *actual* welfare effects on consumers' suboptimal choices due to imperfect information.

welfare measure that can compare the welfare values for perfectly informed consumers with those for imperfectly informed consumers.

We propose such a welfare measure, and call it *quasi-compensating variation*. For a given parameter vector $\theta = (m, p, \hat{r})$ and for any $A \in \mathbb{R}$, define the following function w:

$$w(A, s; \theta) = u(m + A - ps, h(s, \hat{r})).$$

Moreover, for any triple $(p, \hat{r}, v)$ define a standard expenditure function:

$$e(p, \hat{r}, v) = \min \{z + ps | u(z, h(s, \hat{r})) \geq v\}. \tag{4}$$

Then we define our welfare measure as follows.

Definition. Quasi-compensating variation is the monetary compensation required to make a person indifferent between the choices made at the new information structure (θ_1, F_1) and at the old information structure (θ_0, F_0), evaluated at the new state θ_1 as if the person is perfectly informed both before and after the change. Formally, it is the monetary value QCV such that (i)

$$w(0, s^*(\theta_0, F_0); \theta_0) = w(-\text{QCV}, s^*(\theta_1, F_1); \theta_1),$$

where $s^*(\theta, F) = \arg\max \int u(m - ps, h(s, r)) \, dF(r)$ and (ii) the monetary amount QCV is defined on the basis of optimality given θ_1.

First, note that the definition evaluates two utility levels that arise from the choices made suboptimally due to F and, therefore, one must evaluate them as if consumers were perfectly informed about the true ambient risks $\hat{r}_0$ and $\hat{r}_1$ *both before and after the change*. In one sense, a special case of this definition when only F changes closely parallels the way many authors define the value of information.[8] Many value-of-information studies evaluate the choices made prior to obtaining information against the choices that could have been obtained if the decision maker had access to that information. Such evaluation is made on the basis of perfect or best informed decision makers. In another sense, our definition of QCV is more general because it allows us to evaluate a broad set of non-informational parameter changes (*i.e. m, p* and $\hat{r}$). In essence, if there is a value (either positive or negative) attached to information, the value must be relative to other policy environments (*i.e. m, p* and $\hat{r}$ in our case). If so, then the value of changes in policy parameters should also depend on the degree of imperfect information. Our QCV definition provides for one such measure by evaluating policy changes (either informational or environmental) as if people were perfectly informed both about pre- and post-change states at the time of evaluation (but imperfectly informed when they make self-protective decisions).

Part (ii) of the definition requires that the monetary compensation that equates two utility levels must be based on the optimal level of compensation given the new environment θ_1. This means that QCV can be defined by the expenditure function:

$$\text{QCV} = e(p_1, \hat{r}_1, \hat{v}_1) - e(p_1, \hat{r}_1, \hat{v}_0), \tag{5}$$

where $\hat{v}_0 = w(0, s^*(\theta_0, F_0); \theta_0)$ and $\hat{v}_1 = w(0, s^*(\theta_1, F_1); \theta_1)$.[9]

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[8] In a broad sense, the value of information is defined as the increase in expected utility (or welfare) that results from gaining more or refined information about the distribution of possible outcomes prior to decision-making.

[9] We can analogously define quasi-equivalent variation (QEV) choosing θ_0 as the evaluation point. That is, QEV $= e(p_0, \hat{r}_0, \hat{v}_1) - e(p_0, \hat{r}_0, \hat{v}_0)$. For concreteness of our discussion, we focus on QCV for the reminder of the paper.

Lastly, note that we cannot use an indirect utility function for this definition, because in general, $w(0, s^*(\theta, F); \theta) = u(m - ps^*(\theta, F), h(s^*(\theta, F), \hat{r}))$ is not equal to $v(\theta, F) = \max \int u(m - ps, h(s, r)) \, \mathrm{d}F(r)$. The former is the consumer's "realized" welfare level when she chooses $s^*(\theta, F)$ while the latter is the usual indirect utility function given (θ, F) and gives her "expected" welfare level. It is very important to recognize that, if the consumer is perfectly informed both before and after the environmental change, then QCV precisely coincides with the usual CV, because in such a case we have:

$$\text{QCV} = e(p_1, \hat{r}_1, v(p_1, \hat{r}_1)) - e(p_1, \hat{r}_1, v(p_0, \hat{r}_0)) = m - e(p_1, \hat{r}_1, v(p_0, \hat{r}_0)) = \text{CV}.$$

In this sense, QCV is a generalization of CV to the case of imperfect information. An earlier draft of the paper included a graphical representation of this measure for the case where consumers have degenerate beliefs; it demonstrated that our measure works well graphically in that case. Our current model with non-degenerate beliefs is more general but, unfortunately perhaps, not amenable to graphical analysis.

3. Comparative statics

The purpose of this section is to show how comparative statics obtained from our QCV measure differ from those obtained using the traditional CV measure. To prepare for our main results, let us derive the familiar result that the consumer's marginal willingness to pay for a reduction in ambient risk $\hat{r}$ can be expressed solely in terms of the rate of technical substitution between risk-reduction and self-protection (Courant and Porter, 1981; Berger et al., 1987; Bartik, 1988; Quiggin, 1992). To do so, we simply apply the envelope theorem to the cost minimization program (4) and substitute the first-order condition. Then we obtain

$$\frac{\partial e}{\partial \hat{r}} = -p \frac{h_r}{h_s} > 0. \tag{6}$$

We discuss, in sequence, the welfare impacts of changes in $\hat{r}$ (cleanup policy), in p (pricing policy), in F (information policy), and in $(F, \hat{r})$ or (F, p) jointly (mixed policies). For notational simplicity, we will henceforth drop θ from the arguments of $s^*(\theta, F)$. Whenever we use expressions such as $s^*(F)$ and $s^*(\hat{r})$, it should be understood that θ is implicit in these expression. Furthermore, we use $s^*(\hat{r})$ to denote the consumer's choice based on a degenerate belief such that $Pr_F\{r = \hat{r}\} = 1$.

3.1. Cleanup policy

Previous empirical studies have found that *both* willingness to pay *and* expenditures on self-protection are *increasing in perceived ambient risk levels*.[10] However, using our welfare measure we can establish that WTP must be *decreasing in self-protection levels* (thereby, in perceived ambient risk levels) if $h_{rs} \geq 0$.

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[10] This finding is explicit in Powell (1991) but is implicit in other studies using averting expenditures, contingent valuation, and hedonics (*e.g.* Dickie and Gerking, 1996; Gayer et al., 2000).

Proposition 1. *Given the true ambient risk $\hat{r}_0$, let F_0 and F_1 be consumer beliefs such that $s^*(F_0) < s^*(\hat{r}_0) < s^*(F_1)$. S uppose that $h_{rs} \geq 0$ and $u_{zh} = 0$. Then using quasi-compensating variation, the willingness to pay for a reduction in $\hat{r}$ (i.e. $\hat{r}_0 > \hat{r}_1$) has the following relationship:*

$$\mathrm{WTP}^r_{F_0} > \mathrm{WTP}^r_{\mathrm{informed}} > \mathrm{WTP}^r_{F_1}, \tag{7}$$

provided that the reduction is sufficiently small that $s^*(F_0) < s^*(\hat{r}_1) \leq s^*(\hat{r}_0)$ holds.

Proof. Let us first consider the perfectly informed consumer. By definition,

$$\mathrm{CV} = \mathrm{QCV} = \int_{\hat{r}_1}^{\hat{r}_0} \frac{\partial e}{\partial \hat{r}} \, \mathrm{d}\hat{r} = \int_{\hat{r}_1}^{\hat{r}_0} -p \frac{h_r(s(\hat{r}), \hat{r})}{h_s(s(\hat{r}), \hat{r})} \, \mathrm{d}\hat{r}.$$

To find QCV for a consumer with F_0, let us define

$$L(\hat{v}, \hat{r}_1) = z + ps - \lambda[\hat{v} - u(z, h(s, \hat{r}_1))]. \tag{8}$$

Using (5), we can write

$$\mathrm{QCV} = e(p, \hat{r}_1, \hat{v}_1) - e(p, \hat{r}_1, \hat{v}_0) = L(\hat{v}_1, \hat{r}_1) - L(\hat{v}_0, \hat{r}_1) = \int_{\hat{v}_0}^{\hat{v}_1} \frac{\partial L}{\partial \hat{v}} \, \mathrm{d}\hat{v} = \int_{\hat{r}_1}^{\hat{r}_0} -\frac{\partial L}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \hat{r}} \, \mathrm{d}\hat{r},$$

where $\hat{v}_i = u(m - ps^*(F_0), h(s^*(F_0), \hat{r}_i))$ for $i = 1, 2$ and the last equality follows from the change of variable. By the envelope theorem,

$$-\frac{\partial L}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \hat{r}} (\hat{v}, \hat{r}_1) = \lambda(\hat{v}) \frac{\partial[u(m - ps^*(F_0), h(s^*(F_0), \hat{r}))]}{\partial \hat{r}}$$
$$= \lambda(\hat{v}) u_h(s^*(F_0), \hat{r}_1) h_r(s^*(F_0), \hat{r}_1). \tag{9}$$

From the first-order condition with respect to s given $(\hat{v}, \hat{r}_1)$, we obtain $\lambda(\hat{v}) = -p/u_h h_s|_{(s^*(\hat{r}_1), \hat{r}_1)}$. Substituting this into (9), we have

$$-\frac{\partial L}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \hat{r}} (\hat{v}, \hat{r}_1) = -p \frac{u_h(s^*(F_0), \hat{r}_1) h_r(s^*(F_0), \hat{r}_1)}{u_h(s^*(\hat{r}_1), \hat{r}_1) h_s(s^*(\hat{r}_1), \hat{r}_1)}.$$

Note that we do not need to worry about expectation operators because our QCV evaluates utility changes at the perfect information. Because $u_{hh} < 0$ and $h_s > 0$ by A1 and A2, $s^*(F_0) < s^*(\hat{r}_1)$ implies $u_h(s^*(F_0), \hat{r}_1)/u_h(s^*(\hat{r}_1), \hat{r}_1) > 1$. Thus, if $h_{rs} \geq 0$, it follows that

$$-p \frac{u_h(s^*(F_0), \hat{r}_1) h_r(s^*(F_0), \hat{r}_1)}{u_h(s^*(\hat{r}_1), \hat{r}_1) h_s(s^*(\hat{r}_1), \hat{r}_1)} > -p \frac{h_r(s(\hat{r}_1), \hat{r}_1)}{h_s(s(\hat{r}_1), \hat{r}_1)} > 0.$$

Moreover, because s^* is weakly increasing in $\hat{r}$ by Lemma 1, this inequality holds for $\hat{r} \in [\hat{r}_1, \hat{r}_0]$. That is,

$$-p \frac{u_h(s^*(F_0), \hat{r}) h_r(s^*(F_0), \hat{r})}{u_h(s^*(\hat{r}), \hat{r}) h_s(s^*(\hat{r}), \hat{r})} > -p \frac{h_r(s(\hat{r}), \hat{r})}{h_s(s(\hat{r}), \hat{r})} > 0, \quad \text{for all } \hat{r} \in [\hat{r}_1, \hat{r}_0].$$

By the same logic, we have the reverse relationship for the consumer with F_1:

$$-p \frac{h_r(s(\hat{r}), \hat{r})}{h_s(s(\hat{r}), \hat{r})} > -p \frac{u_h(s^*(F_1), \hat{r}) h_r(s^*(F_1), \hat{r})}{u_h(s^*(\hat{r}), \hat{r}) h_s(s^*(\hat{r}), \hat{r})} > 0, \quad \text{for all } \hat{r} \in [\hat{r}_1, \hat{r}_0].$$

Integrating these over $\hat{r} \in [\hat{r}_1, \hat{r}_0]$, we obtain the desired inequality. $\qquad \square$

From the proof, it is clear that if $h_{rs} < 0$, inequality (7) may not necessarily hold. $h_{rs} < 0$ means that the marginal return to self-protection *decreases* with the ambient risk level. That is, self-protection is more effective at a low pollution level than at a high pollution level. Though there may be a few examples of environmental risks that exhibit $h_{rs} < 0$, the assumption of $h_{rs} \geq 0$ appears to be plausible for many of the environmental risks of interest such as mercury, arsenic, and radon in air or water.

The intuition behind this result can be drawn from the illustrative example considered in Section 1. In the example, it is assumed that the personal treatment device can achieve the low concentration level when concentrations are high. This means that the device is effective when concentrations are high but not useful when they are low. Clearly, this case corresponds to $h_{rs} \geq 0$. As explained earlier, the benefit of the collective cleanup is higher for an underprotecting consumer (the value of avoiding a serious, chronic health condition) than for an informed consumer (the cost of the treatment device). Yet, observed averting expenditures are lower for the underprotecting consumer (zero) than for the informed consumer (the cost of the treatment device).

However, we could consider another case in which this relationship may not hold, that is, where $h_{rs} < 0$. Consider, for example, an air pollutant which causes a severe health effect for all individuals without a defensive measure at both high and low concentration levels. The individuals can avoid the health effect with a defensive measure only when the concentration level is low. This case corresponds to $h_{rs} < 0$. Suppose further that the current concentration level is high. In this case, the welfare value of a change from the high to the low concentration level is exactly the same for those who are perfectly informed and for those who underestimate. If the individual underestimates and thinks the concentration level is low, then she uses the protective measure to avoid this health effect. Given her choice, the change from the high to the low concentration level has a positive welfare value (of saving her from the adverse health effect). If the individual is perfectly informed, then she would not protect at all at the high concentration level because she knows it is ineffective. However, if the concentration level is reduced and she is again well informed of this change, then she would use a protective measure, so that she would avoid the adverse effect. Thus, the change in the concentration level has the same welfare value to both underestimating and informed consumers.[11]

3.2. Pricing policy

Analogously, we can consider a WTP relationship for a pricing policy, by which we mean the regulator subsidies the purchase of self-protective measures. Intuitively, we might expect that

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[11] These illustrative examples are intentionally made simple and certainly do not exhaust all cases with an underestimating consumer. When the consumer overestimates ambient risk, the arguments become more subtle. To illustrate, consider again a microbial contaminant in water. In this example, let us assume that it causes a severe health condition, a mild stomach condition, and no health effects, respectively at high, medium, and low concentration levels. Assume further that a treatment device can reduce the personal exposure to a medium level if the concentration level is high, and to a low level if medium. Suppose that the current ambient concentration is medium. When the consumer overestimates, she buys this device (assuming the cost of this device is not too high). When she is perfectly aware, she may or may not buy the device depending on her preference. Consider the government's cleanup policy that reduces the concentration from the medium to the low level. Its benefit for an overestimating consumer is zero if she is not notified of the concentration level after the cleanup or simply the cost of this device if she is notified. On the other hand, the benefit for a perfectly informed consumer is the value of avoiding mild health conditions if she does not use the device or the cost of this device if she does. Thus, the benefit for the perfectly informed consumer is greater than or equal to that for the overestimating consumer.

those who over-protect would be more sensitive to the pricing policy and thus would experience more welfare gains than those who under-protect. In fact, this will be true if one uses the traditional CV measure, because by the envelope theorem $CV = \partial e / \partial p = s^*$. However, the following proposition shows that such a unidirectional relationship does not necessarily hold here.

Proposition 2. *Given the true ambient risk $\hat{r}$, let F_0 and F_1 be consumer beliefs such that $s^*(F_0) < s^*(\hat{r}) < s^*(F_1)$. Suppose that $u_{zh} = 0$. Then using quasi-compensating variation, the willingness to pay for a reduction in p (i.e. $p_0 > p_1$) has an ambiguous relationship:*

$$\text{WTP}^p_{F_0} \lessgtr \text{WTP}^p_{\text{informed}} \lessgtr \text{WTP}^p_{F_1}.$$

Proof. The proof is related to that of Proposition 1. Using (5) and (8), we write QCV as

$$\text{QCV} = e(p_1, \hat{r}, \hat{v}_1) - e(p_1, \hat{r}, \hat{v}_0) = L(\hat{v}_1, \hat{r}) - L(\hat{v}_0, \hat{r}) = \int_{p_1}^{p_0} -\frac{\partial L}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial p} \, dp.$$

Therefore, for a consumer with F_0, the marginal willingness to pay is given by

$$\begin{aligned}
-\frac{\partial L}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial p} &= -\lambda(\hat{v}) \left[u_z s^* + \{ p u_z - u_h h_r \} \frac{\partial s^*}{\partial p} \right] \\
&= \frac{p}{u_h(s^*(\hat{r}), \hat{r}) h_s(s^*(\hat{r}), \hat{r})} \left[u_z s^*(F_0) + \{ p u_z - u_h h_r \} \frac{\partial s^*(F_0)}{\partial p} \right].
\end{aligned} \tag{10}$$

By assumption, the second-order condition holds globally, so that $\partial[- p u_z + u_h h_s]/\partial s < 0$. Hence, $s^*(F_0) < s^*(\hat{r})$ implies

$$p u_z - u_h h_s |_{(s^*(F_0), \hat{r})} < p u_z - u_h h_s |_{(s^*(\hat{r}), \hat{r})} = 0.$$

Because $\partial s^* / \partial p \leq 0$, the second term in the large bracket in (10) is nonnegative. Moreover, from the first-order condition for a perfectly informed consumer, we have

$$\frac{p u_z}{u_h h_s} \bigg|_{(s^*(\hat{r}), \hat{r})} = 1.$$

Using the fact that $s^*(F_0) < s^*(\hat{r})$, $u_z(m - p s^*(F_0)) < u_z(m - p s^*(\hat{r}))$ given $u_{zh} = 0$. Thus, it follows that

$$0 < \frac{p u_z(s^*(F_0))}{u_h(s^*(\hat{r}), \hat{r}) h_s(s^*(\hat{r}), \hat{r})} < \frac{p u_z}{u_h h_s} \bigg|_{(s^*(\hat{r}), \hat{r})} = 1.$$

On the other hand, CV for the perfectly informed consumer is given by

$$\text{CV} = \int_{p_1}^{p_0} \frac{\partial e}{\partial p} \, dp.$$

By the envelope theorem, we immediately obtain

$$\frac{\partial e}{\partial p} = s^*(\hat{r}) > \frac{p u_z(s^*(F_0))}{u_h(s^*(\hat{r}), \hat{r}) h_s(s^*(\hat{r}), \hat{r})} s^*(F_0).$$

Therefore, the first term and the second term of (10) have opposing signs. Analogously, we can obtain the reverse impacts for a consumer with F_1. This completes the proof. $\quad\square$

The intuition behind this result may be understood as follows. The first term of (10) is essentially the pure impact of the policy to reduce p: if the consumer spends less (more) on self-protection, she gains less (more) from the reduction in p. However, the pricing policy has a secondary effect, which is captured by the second term of (10). That is, regardless of her perception about ambient risk, the consumer would adopt more self-protection in response to the decrease in p, which has a welfare improving (decreasing) impact for those who underestimate (overestimate). Thus, the overall impact of a change in p depends on the relative magnitudes of these two offsetting effects.

3.3. Information policy

Our welfare measure also works well when only F changes. Our definition is similar to the conventional ''value of information'' discussed elsewhere. Moreover, the comparative statics for a change in F using our welfare measure are essentially the same as those of ''increasing risk'' (e.g. Feder, 1977; Meyer and Ormiston, 1983), and are therefore omitted.[12] There are, however, two minor differences: (i) QCV evaluates the changes in utility in reference to the true ambient risk level $\hat{r}$, and (ii) QCV is the money metric of the value of information rather than a simple difference in expected utility. Thus, our QCV measure could potentially be used for elicitation of the monetary value of information in a contingent-valuation/behavior framework.

There are several caveats associated with using QCV in this way. Many authors have examined, both empirically and theoretically, the question of how informational programs affect consumer behavior and, therefore, consumer welfare (e.g. Colantoni et al., 1976; Viscusi et al., 1986; Smith et al., 1988; Smith and Johnson, 1988; Ippolito and Mathios, 1990; Smith and Desvousges, 1990; Madajewicz et al., 2007). Though these studies find that consumers do respond to risk information in a rational and sensible way, there is limited evidence to guide us in modeling a consumer's information acquisition process. Because the regulatory authority cannot be sure about how consumers process the information offered, it can only hope to affect consumers' perceptions (i.e. F) in a favorable manner. For example, in developing countries where villagers may be skeptical about regulatory authorities, it is quite possible that an informational program may influence villagers' beliefs in the opposite direction. Furthermore, it is also conceivable that the cognitive effect of providing accurate information in a closed, in-person setting may be quite different from that of information campaigns via mail, newspapers, or TV programs.

3.4. Mixed policies

The usefulness of a welfare measure depends in part on the coherence and the tractability of the measure when it is applied to a complex setting. In connection with the discussions above, one

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[12] Our QCV measure is not restricted by any type of changes in F, as long as its support lies on $[0, r_{max}]$ (e.g. statewise, first-order, or second-order stochastic dominance). However, to obtain clear-cut comparative statics results, one needs to impose some structure on what represents a change in F. Traditionally, the comparative statics of ''increasing risk'' (in the sense of Rothschild and Stiglitz, 1970) are done by representing a change in F with (i) a change in the cumulative distribution function (CDF approach: e.g. Meyer and Ormiston, 1983) or (ii) a linear (deterministic) transformation of the random variable (transformation approach: e.g. Feder, 1977). For example, using a transformation approach, one can show that a mean-preserving increase in dispersion of the consumer's belief in the sense of Feder (1977) is welfare increasing if the consumer is initially underestimating (i.e. $s^*(F_0) < s^*(\hat{r}_0)$) and welfare decreasing if overestimating (i.e. $s^*(\hat{r}_0) < s^*(F_0)$).

might ask whether our measure is still well-defined when a mixed policy, say, of cleanup and informational programs, is used to improve consumer welfare. We consider two mixed policies of interest: (a) information-cleanup policy, $(F_0, \hat{r}_0) \to (F_1, \hat{r}_1)$ and (b) information-pricing policy, $(F_0, p_0) \to (F_1, p_1)$. Our definition incorporates both cases. The relevant questions here are: (i) whether our measure can be decomposed so that the contribution from each can be computed, at least theoretically, and (ii) whether each component has the appropriate (theoretical) sign.

Proposition 3. *Consider* (a) *information-cleanup policy,* $(F_0, \hat{r}_0) \to (F_1, \hat{r}_1)$ *and* (b) *informa-tion-pricing policy,* $(F_0, p_0) \to (F_1, p_1)$. *Then our QCV measure can be decomposed in the following manner.*

(a) $e(p, \hat{r}_1, \hat{v}(p, \hat{r}_1, F_1)) - e(p, \hat{r}_1, \hat{v}(p, \hat{r}_0, F_0)) = [e(p, \hat{r}_0, \hat{v}(p, \hat{r}_0, F_0)) - e(p, \hat{r}_1, \hat{v}(p, \hat{r}_0, F_0))] + [m - e(p, \hat{r}_0, \hat{v}(p, \hat{r}_0, F_0))] - [m - e(p, \hat{r}_1, \hat{v}(p, \hat{r}_1, F_1))]$,

(b) $e(p_1, \hat{r}, \hat{v}(p_1, \hat{r}, F_1)) - e(p_1, \hat{r}, \hat{v}(p_0, \hat{r}, F_0)) = [e(p_1, \hat{r}, \hat{v}(p_1, \hat{r}, F_1)) - e(p_1, \hat{r}, \hat{v}(p_1, \hat{r}, F_0))] + [e(p_1, \hat{r}, \hat{v}(p_1, \hat{r}, F_0)) - e(p_1, \hat{r}, \hat{v}(p_0, \hat{r}, F_0))]$.

Proof. By (5), the LHS of these expressions represent the welfare values of information-cleanup and information-pricing policies, respectively. Iteratively applying (5) to LHS and manipulating, we can obtain the RHS of these expressions. By canceling terms in the RHS, we see that LHS = RHS. The first bracketed term on the right side of (a) is the change in expenditures resulting from the change in $\hat{r}$ to obtain the same utility level $\hat{v}(p, \hat{r}_0, F_0)$, and thus corresponds to the welfare change due to the cleanup policy. The sum of the second and the third bracketed terms in (a) is the difference in the welfare cost of imperfect information at $\hat{r}_0$ and at $\hat{r}_1$ and, thus, coincides with the welfare change due to information policy. Therefore, both components are positive for positive changes. Similarly, the first bracketed term in the right side of (b) is the welfare change due to information policy (evaluated at $(p_1, \hat{r})$) and the second bracketed term is the welfare change due to pricing policy. Again, both terms can be shown to be positive for positive changes. □

4. A generalization of QCV

Until this section, our analysis has been concerned with the case of deterministic health outcomes. However, our measure is also useful to, and consistent with, the case in which h is itself stochastic as in Shogren and Crocker (1991) and Quiggin (1992). Consumers are unaware or uninformed of ambient environmental risk levels partly because their health impacts are stochastic. For example, it is well known that many toxic contaminants such as PCE and TCE may cause cancer in humans only after they are exposed over a long period of time, whereas every dose or exposure is accompanied by some increased risk of cancer. Consumers often fail to associate their realized health outcomes with their exposure to environmental risks because of this time lag and randomness.

In our setup, randomness means that given exposure to the (observed) ambient risk and the self-protection level, the consumer is unsure about where she will be in the continuum of health outcomes, h.[13] This can be formulated in a manner analogous to Quiggin (1992). Let $\Omega = [0, 1]$

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[13] Randomness can also occur in effectiveness of self-protection and preference orderings (see, for example, Quiggin, 2002). Moreover, other epidemiological factors such as age, sex, genes and co-morbidities may influence the stochastic relationship.

be the state space, with each element $\omega \in \Omega$ being arranged from the worst to the best state of the world. Consider the health outcome function $h(\omega; s, r) : [0, 1] \to H \subset \mathbb{R}$. Assume that h is increasing in ω for all (s, r) and $h_s > 0, h_r < 0$ for each ω. Thus, other things being fixed, an increase in self-protection (ambient risk) decreases (increases) the probability of adverse health outcomes. Given the cumulative distribution function G for ω and the subjective belief F, the consumer solves

$$\max_s \ Eu(m - ps, h(\omega; s, r)) = \int \int u(m - ps, h(\omega; s, r)) \, dG(\omega) \, dF(r).$$

The definition of our welfare measure can be generalized to this stochastic-outcome case as follows.

Definition. Quasi-compensating variation is the monetary compensation required to make a person indifferent *ex ante* between the choices made at the new information structure (θ_1, F_1) and at the old information structure (θ_0, F_0), evaluated at the new state θ_1 as if the person is perfectly informed both before and after the change. Formally, it is the monetary value QCV such that (i)

$$E_\omega w(0, s^*(\theta_0, F_0); \theta_0) = E_\omega w(-\mathrm{QCV}, s^*(\theta_1, F_1); \theta_1),$$

where $\theta \equiv (m, p, \hat{r})$ and $s^*(\theta, F) = \arg \max Eu(m - ps, h(\omega; s, r))$ and (ii) the monetary amount QCV is defined on the basis of optimality given θ_1.

It can be shown that the result of Proposition 1 is intact with this formulation. However, this leads to a more subtle, yet important, question. The above formulation effectively imposes the assumption that consumers' evaluation of the stochastic relationship can be separated from their perceptions about the ambient risk (*e.g.* toxic concentration levels). However, as mentioned above, randomness is possibly one of the main reasons why consumers have incorrect perceptions about exogenous risk levels. At present, we are unable to formally represent, and transmit the effects of, imperfect information through all sources of uncertainty due to its analytical complexity.

5. Relationship to contingent valuation

To understand our welfare measure and to see how it is different from a more conventional measure, it may be useful to think of its empirical implementation in a contingent-valuation framework. We argue that, when consumers face uncertainty regarding $\hat{r}$ and can and do take costly steps to self-protect – the case we consider – contingent valuation cannot yield accurate welfare measures. It is true that in a quality contingent-valuation exercise, subjects should be given the information necessary to understand all aspects of the commodity to be valued. In our case, the information must include the ambient risk level.[14] Indeed, it has been shown that the way in which information is provided can influence contingent-valuation results in a statistically significant manner (*e.g.* Thayer, 1981; Mitchell and Carson, 1989; Bergstrom et al., 1989, 1990; Hoehn and Randall, 2002). But even if comprehensive information is provided, and even if all subjects properly understand it and respond accordingly, the resulting estimate of willingness to pay will now apply only to the sample of people who received the information in the survey.

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[14] We would like to acknowledge the valuable help we received from an anonymous reviewer in clarifying this point.

Extrapolating the results to a larger population in order to achieve a measure of aggregate welfare change is inappropriate: all those who were not part of the sample are still as uninformed about the problem as they were beforehand. It is true that in many contingent-valuation studies, self-protection plays no role in eliciting contingent values in question. However, our claim applies to many environmental hazards such as contaminated water, air pollution, and hazardous waste, for which self-protection matters.

To illustrate, let us take an example of a policy change in $\hat{r}$. For concreteness, readers may think of $\hat{r}$ as a concentration level of, say, mercury in drinking water. A typical question in a contingent valuation study is: "How much would you be willing to pay for a reduction in $\hat{r}$, from $\hat{r}_0$ to $\hat{r}_1$?" However, as previous empirical studies suggest, the individual may have an inaccurate perception F_0 and her WTP is typically a function of F_0. Thus, when evaluating her WTP, the analyst has a choice of giving accurate information $\hat{r}_0$ to the respondent *at the time of the survey*. Note that he cannot give her accurate information *at the time of her self-protection decision*. Therefore, $s^*(F_0)$ has not been changed. Suppose that the analyst gives out accurate information $\hat{r}_0$. We compare two alternative strategies: the first strategy corresponds to the current contingent-valuation practice (when done well) and the second strategy employs our welfare measure.

In the first strategy, he can ask her: "How would you change your self-protection decision now that you know the true concentration level?" In this case, she is allowed to change (at least hypothetically) her self-protection behavior, so that she recognizes $s^*(\hat{r}_0)$. After letting her recognize $s^*(\hat{r}_0)$, he can ask "How much would you be willing to pay for a reduction in $\hat{r}$, from $\hat{r}_0$ to $\hat{r}_1$?" In this strategy, her self-protective choice is adjusted from $s^*(F_0)$ to $s^*(\hat{r}_0)$ and she also knows $s^*(\hat{r}_1)$, *i.e.* what she would do when $\hat{r}_0$ is changed to $\hat{r}_1$. Hence, in the first strategy, the analyst obtains WTP = CV, *i.e.* her WTP at perfect information, because he is simply asking for $u(m - ps^*(\hat{r}_0), h(s^*(\hat{r}_0), \hat{r}_0)) = u(m - \text{WTP} - ps^*(\hat{r}_1), h(s^*(\hat{r}_1), \hat{r}_1))$. As argued above, the problem with this strategy is that those who do not participate in the survey would be left imperfectly informed, and the contingent values obtained from the sample (who now have perfect information) would not represent those of the population.

In the second strategy, the analyst does not allow her to consider changing her behavior optimally according to the true information $\hat{r}_0$ or $\hat{r}_1$. Hence, her self-protection is fixed at the pre-evaluation level $s^*(F_0)$. The analyst must let her recognize how her welfare is affected by her misperception. Ideally, the analyst must ask her to evaluate her exposure to health risks associated with $s^*(F_0)$. After letting her recognize $h(s^*(F_0), \hat{r}_0)$ and $h(s^*(F_0), \hat{r}_1)$, he asks her "Given your choice $s^*(F_0)$, how much would you be willing to pay for a change from $\hat{r}_0$ to $\hat{r}_1$ (or alternatively, from $h(s^*(F_0), \hat{r}_0)$ to $h(s^*(F_0), \hat{r}_1)$)?" Clearly, this second strategy corresponds to our QCV when only $\hat{r}$ changes. Note that because the analyst needs to evaluate her WTP at accurate information $\hat{r}$, whichever policy variable, $\hat{r}$, p, or F changes, the true information $\hat{r}$ needs to be presented to the individual if a contingent valuation framework is used. The key to using our QCV measure properly is to recognize the difference between the information or belief F on which the individual's prior self-protection decision is based and the information $\hat{r}$ on which the individual's welfare evaluation is based.

6. Conclusion

Conventional wisdom suggests that an optimal policy equates the aggregate marginal benefit with the aggregate marginal cost of publicly mandated risk mitigation. Previous studies have shown that without uncertainty, the marginal benefit of risk reduction equals the cost of self-protection, multiplied by the rate of technical substitution between ambient risk and self-protection. Thus, at an

optimum, the marginal cost of private abatement should equal the marginal cost of public abatement.

With uncertainty, however, Shogren and Crocker (1991, p. 13) countered this conventional argument, arguing that ''empirical efforts which identify marginal rates of substitution with willingness-to-pay are misdirected.'' Using a state-contingent formulation of the stochastic production function, Quiggin (1992) derived a more positive result that the conventional relationship can be obtained, but only under restrictive assumptions, which were subsequently criticized by Shogren and Crocker (1999).

We have emphasized the view that risk communication and information provision are an integral part of environmental risk management. And the regulatory authority must determine the efficient levels of self-protection, public information provision, and public risk reduction efforts jointly. There is no shortage of empirical findings showing that consumers are often uninformed of, and unsure about, the degree of ambient environmental hazard. To the extent that imperfect information is pervasive, and effective public information programs are costly, the regulator needs to make the trade-off between spending money on cleanup and spending money to inform and educate the public. Any risk communication must contain information about the publicly mandated level of risk. Any public risk mitigation must consider the extent of imperfect information in the population. Therefore, an efficient outcome is achieved only in consideration of self-protection, public information provision, and public risk reduction jointly.

Our paper offers a sound theoretical basis on which to approach this issue. We have shown that neither the standard stated-preference (contingent valuation) nor the standard revealed-preference (averting expenditures) methods offer reasonable estimates in this endeavor. This is because the welfare values of reducing risk may be affected in a subtle way by the marginal cross effect h_{rs} and the extent of imperfect information, either overestimating or underestimating. In contingent valuation, the sample selected for a survey (who is given perfect information by the survey design) does not represent the population (who may have highly heterogeneous information sets). When risks are endogenous in self-protection (therefore, in information structures), extrapolating the results to a larger population in order to achieve a measure of aggregate welfare change is inappropriate: all those who were not part of the sample are still as uninformed about the problem as they were beforehand. In revealed-preference methods, the influence of imperfect information can be more prominent and the true welfare effects may exhibit an inverse relationship to the estimated values if $h_{rs} \geq 0$.

We note two caveats to using our welfare measure. First, cost-benefit analysis using QCV may, though not always, assign high welfare values of reducing risks to ignorant consumers.[15] Resulting mitigation policies may reduce the incentive for consumers to become informed and avert. While this is a noteworthy point, it is not convincing to argue for a policy that punishes uninformed consumers more severely than informed consumers. If members of a subpopulation are less informed and do not self-protect because they are disadvantaged, say, due to low incomes, low education levels, or language barriers, then it becomes crucial to ask whether it is justifiable to rely on self-protection and possibly to further disadvantage the disadvantaged. We still need to evaluate which of the inefficiencies (*i.e.* inefficiency due to reducing incentives to become informed and avert or inefficiency due to imposing high welfare costs on uninformed

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[15] Two conditions must be met for this statement to be true: uninformed consumers underestimate environmental risks and $h_{rs} \geq 0$. Imperfectly informed consumers may either underestimate or overestimate environmental risks (*e.g.* Gayer et al., 2000; Walker et al., 2006).

consumers) is greater. Basic questions of fairness might also come into play. Second, the use of our welfare measure in the contingent-valuation setup may place high cognitive demands on the survey respondents. More practical empirical strategies may need to be developed.

Acknowledgements

Appendix A. Proof of Lemma 1

To see (i), pick any $\lambda \in (0,1)$ and arbitrary two points $(z_0, s_0), (z_1, s_1)$. Suppose that $u(z_0, s_0; r) \geq u(z_1, s_1; r)$. Then,

$$u(z_\lambda, s_\lambda; r) = u(z_\lambda, h(s_\lambda, r)) > u(z_\lambda, \lambda h(s_0, r) + (1-\lambda)h(s_1, r)) > u(z_1, h(s_1, r))$$

$$= u(z_1, s_1; r),$$

where $z_\lambda = \lambda z_0 + (1-\lambda)z_1, s_\lambda = \lambda s_0 + (1-\lambda)s_1$. The first inequality follows from concavity of h in s and from strict monotonicity of u in h. The second inequality follows from strict quasi-concavity of u in (z, h), because

$$u(z_0, h(s_0, r)) \geq u(z_1, h(s_1, r)) \Rightarrow u(z_\lambda, h_\lambda) > u(z_1, h(s_1, r)),$$

where $h_\lambda = \lambda h(s_0, r) + (1-\lambda)h(s_1, r)$.

To show (ii), suppose by contradiction that two indifference curves cross. Then there must exist some point (z^*, s^*) such that $u(z^*, s^*; r_0) = u(z^*, s^*; r_1)$. But, by A2, u is strictly decreasing in r on $[0, r_{max}]$. Thus, at (z^*, s^*), we must have $u(z^*, s^*; r_0) \neq u(z^*, s^*; r_1)$, a contradiction.

The proof of (iii) is a straightforward application of Berge's Maximum Theorem. For a fixed price p of the composite good s, note that $u : \mathbb{R}_+^2 \times [0, r_{max}] \to \mathbb{R}$ is jointly continuous in (z, s, r). The budget constraint is a constant, nonempty, compact-valued correspondence in r: $\Gamma(r) = \{(z, s) \in \mathbb{R}_+^2 : z + ps \leq m\}$. Therefore, the Maximum Theorem applies. Moreover, by (i), u is strictly quasi-concave in (z, s). Because the budget constraint forms a convex set in the (z, s)-space, the maximal solution $s^*(r)$ is unique for each r. Thus, s^* is continuous in r.

Showing (iv) is not so trivial. Let us suppose that u is homothetic in (z, h) and $h_{rs} \geq 0$. Let $r_0 > r_1$. Consider two cases: (a) $s^*(r_0) > 0$ and (b) $s^*(r_0) = 0$. In case (a), if $s^*(r_1)$ is a boundary solution, *i.e.* $s^*(r_1) = 0$, then $s^*(r_0) > s^*(r_1)$, so we are done. If $s^*(r_1)$ is also an interior solution, then both optima satisfy the tangency condition:

$$\frac{u_h(z^*, h(s^*, r))}{u_z(z^*, h(s^*, r))} = \frac{p}{h_s(s^*)}.$$

Suppose by contradiction that s^* is decreasing in r. Then, $r_0 > r_1$ implies $s^*(r_0) < s^*(r_1)$. Then, the RHS of this FONC is

$$\frac{p}{h_s(s^*(r_0), r_0)} < \frac{p}{h_s(s^*(r_1), r_1)}$$

because strict concavity of h implies h_s is weakly decreasing in s. On the other hand, clearly $h(s^*(r_0), r_0) < h(s^*(r_1), r_1)$ and $m - ps^*(r_0) = z^*(r_0) > z^*(r_1) = m - ps^*(r_1)$. This implies that an optimal point moves to the southeast direction in the (z, h)-space. However, homotheticity of u in (z, h) and A1 together imply that the marginal rate of substitution must decrease for a move in this direction:

$$\frac{u_h(z^*(r_0), h^*(r_0))}{u_z(z^*(r_0), h^*(r_0))} \geq \frac{u_h(z^*(r_1), h^*(r_1))}{u_z(z^*(r_1), h^*(r_1))},$$

a contradiction. In case (b), suppose by contradiction $s^*(r_1) > 0$. Then the tangency condition is satisfied at this optimum. Combine this with the FONC for $s^*(r_0) = 0$ to obtain

$$\frac{u_h(z^*(r_0), h^*(r_0))}{u_z(z^*(r_0), h^*(r_0))} \leq \frac{p}{h_s(s^*(r_0), r_0)} < \frac{p}{h_s(s^*(r_1), r_1)} = \frac{u_h(z^*(r_1), h^*(r_1))}{u_z(z^*(r_1), h^*(r_1))},$$

where the second inequality follows, because $0 = s^*(r_0) < s^*(r_1)$. However, the same argument as in (a) implies:

$$\frac{u_h(z^*(r_0), h^*(r_0))}{u_z(z^*(r_0), h^*(r_0))} \geq \frac{u_h(z^*(r_1), h^*(r_1))}{u_z(z^*(r_1), h^*(r_1))},$$

a contradiction. Now consider the case in which $u_{zh} = 0$ and $h_{sr} \geq 0$. Let $L = u(m - ps, h(s, r))$. The first-order condition for interior solutions is $L_s = -pu_z + u_h h_s = 0$. Totally differentiating this, one obtains the explicit expression for

$$\frac{\mathrm{d}s}{\mathrm{d}r} = -\frac{L_{sr}}{L_{ss}}.$$

Under our assumptions, it is clear that $L_{ss} \leq 0$. Thus, we only need to show $L_{sr} \geq 0$, which follows because $L_{sr} = -pu_{zh}h_r + u_{hh}h_r h_s + u_h h_{sr} \geq 0$ if $u_{zh} = 0$ and $h_{sr} \geq 0$.

References

Abdalla, C.W., Roach, B.A., Epp, D.J., 1992. Valuing environmental quality change using averting expenditures: an application to groundwater contamination. Land Economics 68, 163–169.

Abrahams, N.A., Hubbell, B.J., Jordan, J.L., 2000. Joint production and averting expenditure measures of willingness to pay: do water expenditures really measure avoidance costs? American Journal of Agricultural Economics 82, 427–437.

Bartik, T.J., 1988. Evaluating the benefits of non-marginal reductions in pollution using information on defensive expenditures. Journal of Environmental Economics and Management 15, 111–127.

Berger, M., Blomquist, G., Kenkel, D., Tolley, G., 1987. Valuing changes in health risk: a comparison of alternative measures. Southern Economic Journal 53, 967–983.

Bergstrom, J.C., Stoll, J.R., Randall, A., 1989. Information effects in contingent markets. American Journal of Agricultural Economics 71 (3), 685–691.

Bergstrom, J.C., Stoll, J.R., Randall, A., 1990. The impact of information on environmental commodity valuation decisions. American Journal of Agricultural Economics 72 (3), 614–621.

Bockstael, N.E., McConnell, K.E., 1983. Welfare measurement in the household production framework. American Economic Review 73 (4), 806–814.

Bresnahan, B.W., Dickie, M., 1995. Averting behavior and policy evaluation. Journal of Environmental Economics and Management 29, 378–392.

Chien, Yu.-L., Huang, C.J., Shaw, D., 2005. A general model of starting point bias in double-bounded dichotomous contingent valuation surveys. Journal of Environmental Economics and Management 50, 362–377.

Colantoni, C.S., Davis, O.A., Swaminuthan, M., 1976. Imperfect consumers and welfare comparisons of policies concerning information and regulation. Bell Journal of Economics 7 (2), 602–615.

Collins, A.R., Steinback, S., 1993. Rural household response to water contamination in West Virginia. Water Resources Bulletin 29, 199–209.

Courant, P.N., Porter, R.C., 1981. Averting expenditure and the cost of pollution. Journal of Environmental Economics and Management 8, 321–329.

Crocker, T.D., Shogren, J.F., Turner, P.R., 1998. Incomplete beliefs and nonmarket valuation. Resource and Energy Economics 20 (2), 139–162.

Dickie, M., Gerking, S., 1996. Formation of risk beliefs, joint production and willingness to pay to avoid skin cancer. Review of Economics and Statistics 78 (3), 451–463.

Feder, G., 1977. The impact of uncertainty in a class of objective functions. Journal of Economic Theory 16, 504–512.

Gayer, T., Hamilton, J.T., Viscusi, W.K., 2000. Private values of risk tradeoffs at superfund sites: housing market evidence on learning about risk. Review of Economics and Statistics 82 (3), 439–451.

Harrison, D., Rubinfeld Jr., D.L., 1978. Hedonic housing prices and the demand for clean air. Journal of Environmental Economics and Management 5, 81–102.

Hoehn, J.P., Randall, A., 2002. The effect of resource quality information on resource injury perceptions and contingent values. Resource and Energy Economics 24, 13–31.

Ippolito, P.M., Mathios, A.D., 1990. Information, advertising and health choices: a study of the cereal market. RAND Journal of Economics 21 (3), 459–480.

Kim, H.J., Cho, Y., 2002. Estimating willingness to pay for reduced copper contamination in Southwestern Minnesota. Journal of Agricultural and Resource Economics 27 (2), 450–463.

Kim, C.W., Phipps, T.T., Anselin, L., 2003. Measuring the benefits of air quality improvement: a spatial hedonic approach. Journal of Environmental Economics and Management 45, 24–39.

Laughland, A.S., Musser, L.M., Musser, W.N., Shortle, J.S., 1993. The opportunity cost of time and averting expenditures for safe drinking water. Water Resources Bulletin 29 (2), 291–299.

Madajewicz, M., Pfaff, A., van Geen, A., Graziano, J., Hussein, I., Momotaj, H., Sylvi, R., Ahsan, H., 2007. Can information alone change behavior? Response to arsenic contamination of groundwater in Bangladesh. Journal of Development Economics 84, 731–754.

Meyer, J., Ormiston, M.B., 1983. The comparative statics of cumulative distribution function changes for the class of risk averse agents. Journal of Economic Theory 31, 153–169.

Mitchell, R.C., Carson, R.T., 1989. Using Surveys to Value Public Goods: The Contingent Valuation Method. Resources for the Future, Washington, DC.

Murdock, J.C., Thayer, M.A., 1988. Hedonic price estimation of variable urban air quality. Journal of Environmental Economics and Management 15, 143–146.

Murdock, J.C., Thayer, M.A., 1990. The benefits of reducing the incidence of nonmelanoma skin cancers: a defensive expenditures approach. Journal of Environmental Economics and Management 18, 107–119.

Portney, P.R., 1981. Housing prices, health effects, and valuing reductions in risk of death. Journal of Environmental Economics and Management 8, 72–78.

Powell, J.R., 1991. The value of groundwater protection: measurement of willingness-to-pay information, and its utilization by local government decisionmakers. Unpublished Doctoral Dissertation. Cornell University.

Quiggin, J., 1992. Risk, self-protection and ex ante economic value—some positive results. Journal of Environmental Economics and Management 23, 40–53.

Quiggin, J., 2002. Risk and self-protection: a state-contingent view. Journal of Risk and Uncertainty 25, 133–145.

Rothschild, M., Stiglitz, J.E., 1970. Increasing risk, I: a definition. Journal of Economic Theory 2 (3), 225–243.

Shogren, J.F., Crocker, T.D., 1991. Risk, self-protection, and ex ante economic value. Journal of Environmental Economics and Management 20 (1), 1–15.

Shogren, J.F., Crocker, T.D., 1999. Risk and its consequences. Journal of Environmental Economics and Management 37 (1), 44–51.

Smith, V.K., Desvousges, W.H., 1990. Risk communication and the value of information: radon as a case study. Review of Economics and Statistics 72 (1), 137–142.

Smith, V.K., Desvousges, W.H., Fisher, A., Johnson, F.R., 1988. Learning about Radon's risk. Journal of Risk and Uncertainty 1, 233–258.

Smith, V.K., Johnson, F.R., 1988. How do risk perceptions respond to information? The case of Radon. Review of Economics and Statistics 70 (1), 1–8.

Thayer, M.A., 1981. Contingent valuation techniques for assessing environmental impacts: further evidence. Journal of Environmental Economics and Management 8, 27–44.

U.S. General Accounting Office, 1992. Drinking Water: Consumers Often Not Well-informed of Potentially Serious Violations, Report #GAO/RCED-92–135.

Viscusi, W.K., Magat, W.A., Huber, J., 1986. Informational regulation of consumer health risks: an empirical evaluation of hazard warnings. RAND Journal of Economics 17, 351–365.

Walker, M., Shaw, D., Benson, M., 2006. Arsenic consumption and health perceptions in a rural Western U.S. area. Journal of the American Water Resources Association 42, 1363–1370.