Emissions Trading, Firm Heterogeneity, and Intra-industry Reallocations in the Long Run

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Abstract: Design of environmental regulation has substantial implications for size distribution and mass of firms within and across industries in the long run. In a general equilibrium model that accounts for endogenous entry and exit of heterogeneous firms, the welfare impacts of emissions trading are analytically decomposed into the effects on economy-wide income, mass of firms, firm size distribution, output price markups, and factor prices. Distortionary impacts on size distribution and permit price depend on the conditionality of permit distribution, interactions between changes in entry-exit conditions and in aggregate accounting conditions, the factor intensity of entry, and coverage of non-pollution-intensive sectors in emissions trading.

JEL Codes: H23, L11, Q50, Q52, Q58

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Since Rose-Ackerman (1973), economists have long been concerned with the implications of environmental regulations for the long-run industry dynamics. Conventional wisdom in the earlier literature suggests that emissions tax and (auctioned)
emissions trading policies induce efficient allocations in the long run, whereas abatement subsidy and uniform emissions standard policies would distort the entry-exit conditions and induce excessive entry (Polinsky 1979; Spulber 1985; Baumol 1988).

A point of departure for our analysis comes from two observations we make about Spulber’s seminal paper (1985), which showed that a uniform emissions standard would induce excessive entry and thus inefficiency under perfect competition with identical firms. This result occurs because the emissions standard confers firms the right to pollute up to the standard upon entry, which also serves as an entry subsidy. Our first observation is that because Spulber assumes identical firms, excessive entry in his context simply means a larger number (or mass) of firms in the long-run equilibrium. When firms are heterogeneous, however, excessive entry could mean either entry of less productive firms, a larger mass of firms, or both. Indeed, an increasing number of empirical studies have substantiated the existence of large and persistent variation in firm-level productivity across firms (e.g., Cabral and Mata 2003; Eaton, Korum, and Kramarz 2011). When firms are heterogeneous, environmental regulations might affect different firms differently both at intensive and extensive margins, inducing changes in the size distribution of firms.

Second, the policy-induced effect on the extensive margin alone (i.e., on size distribution and mass of firms) may have a second-order impact on the intensive margin, via its effect on competition in the commodity and factor markets. Since Gibrat’s (1931) seminal work, an extensive body of literature (e.g., Simon and Bonini 1958; Lucas 1978; Cabral and Mata 2003; Luttmer 2007) has investigated the economic mechanisms underlying the size distribution of firms that is often observed to be stable and approximately Pareto or lognormal. Their motivation comes from the idea that the size distribution alone may have important implications for consumer welfare, industry competition, and antitrust regulations. Recently, economists (e.g., Melitz 2003; Eaton et al. 2011) have examined trade-induced variations in the size distribution of firms. In the environmental economics literature, a recent empirical study by Greenstone, List, and Syverson (2012) on US manufacturing firms shows that environmental regulations induced exit of less productive firms, causing the industry to be more concentrated, yet decreased average productivity of firms. Presumably, this occurs because those productive firms that stay in the industry produce substantially less due to the cost of environmental regulations. Such a policy-induced change in the size distribution of firms creates intra-/inter-industry reallocations of firm-level variables. Hence, the overall impacts of environmental regulations on aggregate variables of interest such as permit price, output, and welfare would be
determined through intricate interactions between their effects on the extensive margin and on the intensive margin.

This paper proposes a theoretical framework that enables us to disentangle these intricate effects of environmental regulations on the size distribution and mass of firms in a general equilibrium model that accounts for entry and exit of heterogeneous firms. To this end, we focus on the design issues of emissions trading (ET). In first-best settings, a successful ET policy should make the initial distribution of emissions allowances unconditional on all relevant economic decisions by the regulated firms such as emissions, output, or entry. However, conditional allocation rules have often been used in practice in order to protect certain industries or to alleviate preexisting market distortions. For example, the European Union Emission Trading Scheme (EUETS) has the new entrant and closure provision under which firms lose their permits upon exit (Ellerman and Buchner 2007). The Waxman-Markey legislation proposed an output-based allocation (OBA) rule where firms receive emissions allowances proportional to their output levels. When permit allocation is conditioned on entry or production, however, firms receive a de facto entry/production subsidy, which may alter firms’ pollution-generating activities both at intensive (i.e., production/abatement) and extensive margins (i.e., entry/exit). In a recent paper, Hahn and Stavins (2011) point out that such conditional distribution of permits is indeed one of the six ways in which the “independence property” of emissions trading can fail.1

We start with the Melitz-type economy (2003) consisting of a continuum of heterogeneous firms. In the model, the firms make endogenous entry, draw productivity levels from a common distribution upon entry, and then produce in a monopolistically competitive industry using two inputs, labor and emissions, in a manner analogous to Copeland and Taylor (1994). The model then embeds a suit of conditional allocation rules under the ET policy. As in Melitz (2003) and other related studies, our analysis is restricted to comparison of stationary equilibria, wherein the distribution of all firm-level variables stays constant and firms form perfectly rational expectations about all the industry-level variables (including the price of permits) when making all relevant decisions. The advantage of this approach is its tractability, in particular with respect to the policy-induced effects on both the intensive and extensive margins.

We consider several permit allocation rules, all of which have been applied in practice and received significant attention in previous studies. These are, in the order of increasing latitude of conditionality: (i) auctioning, (ii) grandfathering with a permanent allocation rule (as in the US Acid Rain Program), (iii) grandfathering with

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1. In the literature, the independence property of emissions trading is defined: the emissions market equilibrium minimizes the total cost of abatement given the emissions cap and the equilibrium allocation of permits is independent of the initial distribution of permits (Hahn and Stavins 2011).
an entry/closure provision (as in the European Union Emissions Trading System),
and (iv) grandfathering with an output-based allocation (OBA) rule (as discussed in
previously proposed US legislations). Considering these schemes one by one allows
us to disentangle the equilibrium effects of each allocative design. For instance, we
demonstrate that while rule iii has a direct impact on firms’ decisions at the extensive
margin (i.e., entry and exit), rule iv influences those at both the intensive and exten-
sive margins.

This paper contributes to four areas of research. First, the paper adds to the
body of literature that has investigated linkages between environmental regulation
and competitiveness of the manufacturing industry (see Jaffe et al. [1995] and
Ambec et al. [2013] for extensive reviews). Empirical studies illustrate the impact
of US environmental regulation on firms’ output, productivity, and exit decisions in
the manufacturing sector (e.g., Greenstone et al. 2012; Ryan 2012). While these
studies find convincing evidence for the causal linkages between environmental reg-
ulation and industry performance, the underlying economic mechanisms that induce
changes in the size distribution of regulated firms still remain unclear—an aspect
that the literature in industrial organization and international trade have found to
play a crucial role in determining industry performance. Our paper offers a theoreti-
cal foundation to fully explain the mechanisms and shows that the impact of emis-
sions trading on the size distribution and the average firm profits depends on a
number of factors: the conditionality of permit distribution, interactions between
changes in entry-exit conditions and in aggregate accounting conditions, the factor
intensity of entry, and coverage of non-pollution-intensive sectors in emissions trad-
ing. The proposed model could be readily extended to other types of environmental
regulations such as emissions tax/subsidy and command-and-control policies.

Second, this paper complements a line of studies that incorporate Melitz’s framework
in analyzing the impacts of trade liberalization on pollution (Kreickemeier and Richter
2014) or of environmental regulations on firms’ exports and emissions (Yokoo 2009;
Cui, Lapan, and Moschini 2012; Konishi and Tarui 2013). Kreickemeier and Richter
(2014) assume a constant emissions rate per unit of output. Yokoo (2009) assumes a
Copeland-Taylor framework in modeling firms’ variable emissions rates. Cui et al.
(2012) also use the Copeland-Taylor framework but augment it by incorporating firms’
binary technology choice. However, neither of these studies considers implications of
grandfathering schemes, either for empirical implementation or for welfare analysis. As demonstrated in the paper, specific design features of grandfathered emissions trading have substantial implications for both entry-exit and aggregate accounting conditions in the Melitz-type economy, whose impacts can be analytically decomposed into five competing effects on economy-wide income, mass of firms, size distribution, price markup, and factor price. Because many existing emissions markets use grandfathering schemes in practice, these policy-induced differences may have important implications for identification and estimation in empirical studies and, therefore, can motivate future empirical studies. Konishi and Tarui (2013) investigate the intra-industry impacts of emissions tax and emissions trading policies in a model analogous to the present paper. However, they focus on the relative performance of different policy instruments and do not fully explore implications of the emissions cost in either the fixed input of production or entry. Our paper instead focuses on the design issues of emissions trading, and shows that different assumptions on the fixed input of production or entry yield different theoretical predictions on the productivity cutoffs, which can be of empirical importance.

Third, there is a growing body of literature that has investigated the effects of conditional allocation rules in second-best settings with preexisting distortions theoretically (Jensen and Rasmussen 2000; Fischer and Fox 2007) and empirically (Dardati 2013; Fowlie, Reguant, and Ryan, forthcoming). Fischer and Fox (2007) use a computational general equilibrium model to investigate the implications of allocation rules for domestic rebate programs in a static context. They find that an auctioned emissions trading outperforms an OBA rule in terms of social welfare, with a permit price under the auctioned system roughly equal to that under the OBA. Their focus is, however, not on the long-run equilibrium impacts, and hence, they do not address the allocative effects on either the intra-industry firm distribution or the mass of firms in the long run. Dardati (2013) considers a Hopenhayn-type model of heterogeneous firms under perfect competition. Using the model calibrated with the data from US electric power plants, she finds that the exit rate of plants would have been 7% lower under the closure provision of the EU Emissions Trading System than under the permanent allocation rule of US Acid Rain program. Though ours is a Melitz-type model of heterogeneous firms under monopolistic competition, this finding of hers is indeed consistent with our model's prediction that the overall mass of firms is larger under the closure provision than under the permanent allocation rule. Fowlie et al. (forthcoming) use a dynamic partial-equilibrium model of an oligopolistic industry to empirically investigate the effects of alternative allocation rules in the US cement industry. They find that intensity” and that “export status is negatively correlated with emissions intensity” (Cui et al. 2012, 1). Exploring such a channel is left for our future research.
dynamically updating permit allocations in proportion to production in the previous period does better than auctioning, for such an allocation rule can mitigate distortions from both emissions leakages and market power in the commodity market. Their approach takes into account firms’ dynamic responses in discrete technology investments to policy designs. Importantly, however, they assume a constant price of permits with a flat permit supply curve in the neighborhood of the cap, assuming that the cement industry is small relative to the overall emissions market. In contrast, ours is a general equilibrium analysis, with a vertical permit supply and an endogenous permit price. Though much of our paper is organized around a one-sector model, our analysis can be readily extended to multiple sectors where such an assumption is more useful (see sec. 7). In this sense, our model is more general in its scope yet is substantially more tractable than these studies, allowing us to address the size distribution and mass of firms in the long-run equilibrium—a gap in the literature we attempt to fill in.

Fourth, there exists a large body of literature that has investigated the distortionary effects of environmental regulations on entry-exit behavior in a variety of setups (e.g., Carlton and Loury 1980; Collinge and Oates 1982; Kohn 1985; Spulber 1985; McKitrick and Collinge 2000; Pezzey 2003). However, we are not aware of studies of the entry-exit problem that have explicitly considered heterogeneity of firms. Our analysis suggests that when firms are heterogeneous, there is a subtle and important interaction between the distortion on entry-exit conditions and that on aggregate resource constraints (or equivalently, between the size distribution of firms and the mass of firms)—a pathway that can motivate future empirical and theoretical works.

Our results, however, rest on two qualifying assumptions of the model, which present both advantages and disadvantages over existing studies. The first is the assumption of monopolistic competition. Much of the existing literature on the theory of environmental regulations assumes either perfect competition or oligopolistic competition because pollution-intensive industries such as cement, iron and steel natural gas, and nonferrous metals have been traditionally perceived as homogeneous-good industries. However, at least some of these industries have increasingly become differentiated-good industries with substantial evidence of intra-industry trade. Dispersion measures of firm size within a sector in the United States, which “captures the joint effect of the dispersion of firm productivity and the elasticity of substitution” (Helpman, Melitz, and Yeaple 2004, 307) are 1.48 for stone, minerals, and ceramics, 1.88 for ferrous metals, and 1.49 for nonferrous metals—these numbers are roughly comparable to some of the well-known monopolistically competitive industries such as textiles (1.84) and apparel (1.57). Furthermore, some of the well-known differentiated-good industries such as chemical are also pollution intensive. For instance, the organic and inorganic chemical industry accounts for 9.7%, 18.7%, 16.5%, 7.2%, 12.7%, and 11.1% of the total emissions from all US manufacturing processes in 1999 for CO, NH₃, NOₓ, PM10, SO₂, and VOC, respectively. These
numbers are not small compared to those for the iron and steel industry, which accounts for 28.6%, 13.9%, 7.3%, 13.7%, 5.6%, and 4.4%, respectively (EPA 2005). Moreover, after accounting for emissions from purchased electricity, the chemical industry is the second-largest source of combustion-based industrial greenhouse gas emissions in the United States, accounting for 18% of combustion-based total industrial emissions in 2002, whereas the cement and the iron and steel industries only account for 4% and 6%, respectively (EPA 2008).³ The Melitz-type economy is known to yield theoretical predictions that are roughly consistent with empirical regularities in manufacturing industries, including these pollution-intensive industries (e.g., Helpman et al. 2004; Eaton et al. 2011).

The second qualification is the full-employment assumption. Our model is a general equilibrium model, and we explicitly use this assumption in deriving the mass of firms and the price of permits (but not the cutoff productivity). Indeed, an important contribution of the paper is this explicit account of the aggregate resource constraints in examining the policy-induced effects. Presumably, however, an introduction of emissions trading would cause reallocation of employment from pollution-intensive industries to less pollution-intensive industries. Hence, the full employment assumption would be more valid in the model incorporating two or more industries with different pollution intensities. Section 6 explores such a model and shows that initial permit distributions to different sectors have important implications for the equilibrium price of permits as well as inter-industry reallocations of employment, emissions, and firms.

The paper is organized as follows. The next section describes our model environment, with auctioned emissions trading as a benchmark. We first describe the equilibrium properties of the model under the auctioned ET in comparison to no regulation in section 2. We then examine the equilibrium properties under grandfathering with entry/closure provision in section 3. Alternative assumptions about the cost of emissions in entry are also discussed there. The output-based allocation is examined in section 4. We then explore welfare implications of our analysis in section 5. A model with multiple sectors is discussed in section 6. The last section concludes.

1. THE MODEL SETUP

1.1. Regulatory Setup
We first touch on the regulatory environment. Let \( Z > 0 \) be a cap on aggregate emissions, which is assumed exogenous to the model (until sec. 5) and stay constant for all periods. The only regulatory variable of interest in this paper, therefore, is the allocation rules on the initial distribution of permits. The rules are an-

³. Similar estimates are also available from US Department of Commerce Economics and Statistics Administration (2010).
nounced once and for all periods, which firms observe prior to all relevant decisions. This approach is identical to that of Melitz (2003) in his analysis of the impact of international trade. In all cases, a continuum of firms participates in the emissions market with undifferentiated permits, so that the emissions market is perfectly competitive. In this section, we describe our benchmark model for the case of auctioned emissions trading. We then examine the impacts of alternative allocation rules one by one in subsequent sections.

1.2. Demand
Consider an economy characterized by both pollution-intensive production and monopolistic competition (e.g., chemical, iron and steel, and nonferrous metals). The preferences of a representative consumer are given by the Dixit-Stiglitz constant elasticity of substitution (CES) utility with an additional disutility from aggregate pollution:

\[
U = \left[ \int_{\omega \in \Omega} q(\omega)^\sigma d\omega \right]^{1/\sigma} - Lb(Z),
\]

where \( \omega \) is an index of commodities, \( \Omega \) the measure of the set of available goods, \( L \) is the population size, and \( h \) is a convex function of aggregate emissions \( Z \). The parameter \( \rho \) defines the elasticity of substitution between commodities \( \sigma \equiv 1/(1-\rho) \).

We assume that \( \rho \in (0, 1) \) (equivalently, \( \sigma > 1 \)): that is, the commodities are substitutes. We also assume that individual consumers ignore the term \( h(Z) \) in making the consumption decision.\(^4\) Then the standard two-step procedure as in Dixit and Stiglitz (1977) yields the following formulas for consumer demand and expenditures:

\[
q(\omega) = Q \left[ \frac{p(\omega)}{P} \right]^{-\sigma} \quad \text{and} \quad r(\omega) = R \left[ \frac{p(\omega)}{P} \right]^{1-\sigma},
\]

where \( r(\omega) = p(\omega)q(\omega) \), \( P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \) is the aggregate price index, \( Q = \left[ \int_{\omega \in \Omega} q(\omega) d\omega \right]^{1/\sigma} \) the aggregate quantity index, and \( R = PQ \) the economy’s total expenditure/income.

1.3. Production and Abatement
As in Melitz (2003), each firm is endowed with productivity \( \phi \in [0, \infty) \) and employs only one input, labor, which is inelastically supplied at the aggregate level \( L \). For expositional ease, higher \( \phi \) represents higher productivity. Unlike in Melitz (2003), firms discharge pollution as a by-product of production. Firms have access

\(^4\) This assumption is justified by assuming that the representative consumer consists of a continuum of consumers.
to abatement technologies, which also use labor to reduce emissions. Following Copeland and Taylor (1994), the joint production function can then be written as

\[ q = \begin{cases} \phi z^\beta l^{1-\beta} & \text{if } z < \lambda l, \\ \phi A l & \text{otherwise,} \end{cases} \tag{3} \]

where \( \lambda > 0 \) is the bound on the substitution possibility between labor and pollution inputs and \( A = \lambda^\beta.5 \) The Cobb-Douglas specification in (3) is innocuous and simplifies much of our subsequent analyses. Virtually all of our results still hold with some minor modifications if we instead assume a CES production function.6

The cost function consists of a variable component as well as a fixed overhead component, both of which are assumed to incur the cost of emissions (e.g., a factory or equipment emits a certain amount of pollution irrespective of the amount of output as long as it is in operation). The fixed component of production is assumed to have the same emissions intensity as the variable component, as in Bernard, Redding, and Schott (2007). The assumption on the emissions intensities is not innocuous and has important implications for our results. We shall revisit this issue in section 1.4.

Under these assumptions, a firm’s cost minimization with respect to both variable and fixed inputs yields the following cost function:

\[ c(q) = \frac{q}{\phi} + \frac{f}{\phi} \tau \beta w^{1-\beta}, \tag{4} \]

where \( \tau > 0 \) is the price of emissions permits, \( f > 0 \) the fixed cost (in terms of the unit of output), and \( w > 0 \) the unit cost of labor, which we normalize to equal 1.7

Given the cost function, input prices, and the inverse residual demand \( p'(q) \) implicitly defined by (2), the firm’s profit maximization yields the optimal markup:

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5. Because output must be bounded above for a given level of labor input, the substitution possibility between labor and pollution must be bounded by some \( \lambda > 0 \). When \( \tau \) is zero (no regulation) or sufficiently low, firms would attempt to substitute more pollution for labor, eventually reaching the maximum substitution possibility. See Copeland and Taylor (1994) for a detailed discussion on this production function.

6. Online appendix B analyzes the case with the CES production function.

7. To be more precise, a firm’s cost minimization along with (3) yields the cost function:

\[ \left( \frac{q}{\phi} + \frac{f}{\phi} \right)^\frac{\tau \beta w^{1-\beta}}{\left( \frac{1}{1-\beta} \right) \tau \phi} = \tau. \]

As in Bernard et al. (2007), we redefine the unit so that the cost function (4) will be used throughout the paper. Furthermore, the firm’s cost-minimizing choice of \( l \) and \( z \) must satisfy \( z = \lambda l \) if input price ratio \( w/\tau \) exceeds the marginal rate of technical substitution along the ray \( z = \lambda l \). With \( w \) normalized to 1, the condition can be written \( \tau \leq \beta/[(1-\beta)\lambda] \). From here on, we assume that \( \tau \) is large enough (the emissions cap \( Z \) is small enough) to induce emissions reduction beyond the no-regulation level: \( \tau > \beta/[(1-\beta)\lambda] \).
\[ p(\phi) = \frac{\tau^\theta}{p\phi}, \quad (5) \]
as well as output \( q(\phi) \), revenue \( r(\phi) \), and variable emissions \( z_{pv}(\phi) \) (the subscript \( pv \) refers to factors used in the variable part of production). It then follows that the ratios of any two firms’ outputs, revenues, and variable emissions can be conveniently expressed as the functions of ratios of their productivity levels under all policy regimes.

\[ \frac{q(\phi_1)}{q(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^\sigma, \quad \frac{r(\phi_1)}{r(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^{\sigma-1}, \quad \frac{z_{pv}(\phi_1)}{z_{pv}(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^{-1}. \quad (6) \]

Thus more productive firms are larger, not only in output and revenues (as in Melitz), but also in variable emissions. Moreover, the ratio of any two firms’ emissions rates (i.e., variable emissions per unit of output) is an inverse of the ratio of the two firms’ productivity levels.

\[ \frac{z_{pv}(\phi_1)}{q(\phi_1)} = \frac{\phi_2}{\phi_1}. \quad (7) \]

Because firms’ fixed emissions do not vary by productivity, these relationships imply that more productive firms emit more in absolute terms, yet emit less per unit of output.

Though it seems quite intuitive that more productive firms tend to be larger in all firm-level variables, it is not necessarily obvious why more productive firms need to have less emissions rates. But this follows directly from the Copeland-Taylor framework. Because firms use emissions as an input for production and because more productive firms can produce more given any input levels, more productive firms emit less for a given output level, including the profit-maximizing output level. Indeed, the empirical evidence suggests that this result is consistent with observed firm behavior (Cole, Eliott, and Shimamoto 2005; Shadbegian and Gray 2006; Mazzanti and Zoboli 2009; Cui et al. 2012).

Last, the variable part of the profit equals \( (p - \tau^\theta/\phi)q \). Therefore, we can rewrite firm’s profit as

\[ \pi(\phi) = \frac{r(\phi)}{\sigma} - f\tau^\theta. \quad (8) \]

Because the firm’s revenue is increasing in \( \phi \), the firm’s profit is also increasing in \( \phi \) per equation (8).

1.4. Entry-Exit Conditions

Prior to entry, each entrant first pays a fixed entry cost. This entry cost represents the unrecoverable cost of intangible and tangible resources devoted to entry such as research and development, learning about the industry, obtaining business licenses,
and clearing environmental assessments. In the base model, we assume this entry activity discharges pollution at the same emissions intensity as production, so that the entry cost takes the following form:

$$fe^\tau \beta w^{1-\beta}$$

(9)

where $w \equiv 1$ as before and $f_e > 0$ is an entry-cost parameter. Though the assumption is consistent with Bernard et al. (2007), by this we are also assuming, first, that firms face the same factor intensities in variable and fixed production costs as well as fixed entry costs, and second, that firms need to buy permits for emissions generated not only through production but also through entry, and third, that the regulatory authority has the monitoring and enforcement capacity to ensure that. We make this assumption not just for expositional ease but also for its economic significance. As will be demonstrated more fully in section 3, factor intensity differences between any of these costs alone can alter the entry-exit and the aggregate accounting conditions regardless of the design of emissions trading. Hence, maintaining the same factor intensity helps us to identify the sources of distortionary impacts that originate from the design of emissions trading. In our companion paper (Konishi and Tarui 2013), we consider a model without a fixed cost of emissions (i.e., $\beta = 0$ in fixed costs of production and entry). We shall also discuss the implications of alternative assumptions on the entry cost in section 3.

After paying the entry cost, the firm observes its productivity level $\phi$, drawn from a common distribution $G$ that has a positive support over $(0, \infty)$ with density $g$. Each successful entrant produces a unique commodity in the commodity market and buy/sell pollution permits in the emissions market. The firms that make negative profits exit the market immediately. Firms then face an exogenous probability $\delta$ of adverse shocks each period that force them to exit the market. Let $\phi^*$ be the cutoff productivity level such that $\pi(\phi^*) = 0$. Using firm’s profit (8), we see that $\pi(\phi^*) = 0$ implies

$$r(\phi^*) = \sigma f^\tau.$$  

(10)

Because $\pi(\cdot)$ is increasing in $\phi$, firms with $\phi < \phi^*$ immediately exit and never produce.

The distribution of incumbent firms then is determined by the initial distribution $G$ of productivity shocks, conditional on successful entry:

$$\mu(\phi) = \begin{cases} \frac{g(\phi)}{1 - G(\phi^*)} & \text{if } \phi \geq \phi^* \\ 0 & \text{otherwise} \end{cases}.$$  

(11)

Hence, the cutoff $\phi^*$ uniquely defines the distribution of firm-level productivity, which also uniquely defines the distributions of all firm-level variables such as emissions, outputs, and revenues. Substituting (10) and (6) in (8) and taking the conditional average of firms’ profits, we see that the average profit $\bar{\pi}$ satisfies
\[ \bar{\pi} = \pi(\bar{\phi}) = \left[ \left( \frac{\bar{\phi}}{\phi^*} \right)^{\sigma - 1} - 1 \right] f \tau^{\theta}, \]  
\text{(ZCP)}

where \( \bar{\phi} \) is the weighted average productivity defined by
\[ \bar{\phi}(\phi^*) = \left[ \int \phi^{\sigma - 1} \mu(\phi) d\phi \right]^{1/(\sigma - 1)}. \]
\text{(12)}

Equation (ZCP) implicitly defines the exit (and shutdown) condition, since it describes the relationship between the cutoff productivity \( \phi^* \) and the average profit \( \bar{\pi} \) implied by firms’ exit behavior.

To pin down the long-run equilibrium, we also need to derive the entry condition. To do so, we follow Melitz (2003) and Bernard et al. (2007), and focus on the stationary (and steady-state) equilibrium in which all aggregate variables as well as the mass and distribution of incumbent firms stay constant over time.\(^8\) The stationary equilibrium concept is useful for our analysis not only because of its tractability but also because it is a dynamic-model analogue of the long-run equilibrium concept employed in the conventional static models of environmental regulations.

Because a potential entrant does not observe its productivity prior to entry, the entrant enters the market if and only if its ex ante expected value of entry is higher than or equal to the fixed cost of entry. In the stationary equilibrium, a successful entrant with productivity \( \phi \) earns \( \pi(\phi) \) and faces the probability of death \( \delta \) in each period, so that its value of entry is equal to \( \sum_{t=0}^{\infty} (1 - \delta)^t \pi(\phi) = \pi(\phi)/\delta \). The ex ante expected value of entry then is \( E[\pi(\phi)/\delta] = p_{\pi_n}(\pi/\delta) \), where \( p_{\pi_n} = 1 - G(\phi^*) \).

Free entry implies that entry should occur until all net expected profits are exhausted. Thus entry should occur until
\[ (1 - G(\phi^*)) \frac{\bar{\pi}}{\delta} - f \tau^{\theta} = 0. \]
\text{(FE)}

Equation (FE) defines the entry condition.

Because equations (ZCP) and (FE) jointly constitute the entry-exit condition, any potential distortions due to the conditional allocation rules should, in principle, appear in these equations. Substituting (ZCP) into (FE), we obtain the equation that governs firms’ entry-exit behavior that must hold in equilibrium:

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\(^8\) The stationary equilibrium concept has been employed in Melitz (2003) and a number of subsequent studies for several reasons. First, the empirical literature finds that the size distribution of firms persists over time. Second, the theoretical literature suggests that a history of firm-specific independent shocks in a dynamic process can generate such a stationary distribution of productivity (e.g., Luttmer 2007). Third, while it is possible to incorporate an evolution of size distributions over time in the spirit of Ericson and Pakes (1995) or Hoppenhaupt (1992), such models tend to be substantially less tractable.
Note that the combined entry-exit condition is solely a function of exogenous parameters of the model, and so is the cutoff productivity $\phi^*$. Hence, the existence and uniqueness of $\phi^*$ are ensured (see Melitz [2003] for the proof).

1.5. Aggregate Conditions

Once the cutoff productivity is determined, aggregate resource constraints must bind the mass of firms (or equivalently, mass of varieties) that can be supported in the stationary equilibrium. Here, we emphasize one significant implication of our model setup—the aggregate resource constraints must hold not only for labor but also for emissions used through all levels of economic activity. The economy is inherently endowed with the labor supply $L$, which must be allocated for use in either abatement, investment by new entrants, or production. The permit supply $Z$ also serves as the economy’s pollution endowment, which must be used in either production or investment by new entrants. (Recall that in the Copeland-Taylor framework, emissions as a by-product of production and investment are translated into an input for production and investment.) As we shall see below, conditional allocation rules may alter these aggregate resource constraints either through distortions in the entry-exit condition or through ways in which the permit supply is accounted for.

To characterize the aggregate conditions under auctioning, assume that, as in Copeland and Taylor (1994) and other related literature, the government recycles back the revenues from auctioned permits, in a lump-sum manner, to the consumers. Then the sum of aggregate payments to labor and pollution permits used in production must equal the difference between the aggregate revenue and the aggregate profit: $L_e + \tau Z_e = R - \Pi$, where the subscript $p$ refers to variables used for production. On the other hand, the sum of aggregate payments to labor and pollution permits used in investment must equal the aggregate cost of entry: $L_e + \tau Z_e = N_f \tau^e$, where $N$ is a mass of new entrants and the subscript $e$ refers to variables used for entry. Because all aggregate variables remain constant in all periods, a mass of successful entrants $(1 - G(\phi^*))N$ must equal the mass of firms $\delta M$ that are hit by adverse shocks. Combining these with (FE), we have:

\[
L_e + \tau Z_e = \frac{\delta M \tau^e}{1 - G(\phi^*)} = M \pi = \Pi.
\]

Hence, $L = L_p + L_e = R - \Pi - \tau Z_p + \Pi - \tau Z_e = R - \tau Z$. In other words, the aggregate income must equal the sum of aggregate payments to labor and pollution permits:

\[
R = L + \tau Z.
\]
Now consider the accounting equation for aggregate emissions: $\tau Z = \tau Z_{pv} + \tau Z_{pf} + \tau Z_e$, where $Z_{pv}$ and $Z_{pf}$ stand for the variable and the fixed parts of aggregate emissions from production, respectively. From individual firms’ optimality conditions, $\tau z_{pv}(\phi) = \rho \beta \tau^\phi$ and $\tau z_{pf} = \beta f \tau^\phi$. Integrating them over all firms, we have $\tau Z_{pv} = \rho \beta R$ and $\tau Z_{pf} = \beta M f \tau^\phi$. Moreover, the Cobb-Douglas specification of the entry cost implies $\tau z_e = \beta \Pi = \beta M \pi$. Hence, we can rewrite this accounting equation as follows.

$$\tau Z = \rho \beta R + \beta M f \tau^\phi + \beta M \pi. \quad (15)$$

Applying $M = R/\tau$ and $\tau = \sigma(\bar{\pi} + f \tau^\phi)$ and canceling terms, we obtain the demand for permits as a function of the economy-wide income:

$$\tau Z = \beta R, \quad (16)$$

which says that the share of the aggregate payments to pollution permits in the aggregate expenditure must equal the emissions intensity.

Using (14) and (16), we obtain the equilibrium price of permits under the auctioned ET:

$$\tau = \frac{\beta L}{(1 - \beta) \bar{Z}} \quad (17)$$

for a given cap on emissions $Z$. Moreover, using $M = R/\tau$ and $\tau = \sigma(\bar{\pi} + f \tau^\phi)$ along with (14), (16), and (ZCP), the equilibrium mass of firms under auctioned ET is given by

$$M = \frac{L}{\sigma (1 - \beta)(\bar{\pi} + f \tau^\phi)}. \quad (18)$$

Note here that because $\bar{\pi}$ depends on average productivity $\bar{\phi}$, $M$ also depends on $\bar{\phi}$.

Once the distribution and the mass of firms are identified, all aggregate variables can be readily determined as follows:

$$P = M^{1/(1 - \sigma)} p(\bar{\phi}), \quad Q = M^{1/\sigma} q(\bar{\phi}), \quad R = PQ = Mr(\bar{\phi}), \quad \Pi = M \pi(\bar{\phi}), \quad Z = M z(\bar{\phi}). \quad (19)$$

These relationships mean, as in Melitz (2003), that because the weighted average of the firm’s productivity levels $\bar{\phi}$ is independent of the number of firms $M$, an industry comprising $M$ with any distribution that yields the same average productivity $\bar{\phi}$ behaves the same way as an industry with $M$ representative firms having the same productivity $\phi = \bar{\phi}$. Hence, the impacts of emissions trading on aggregate variables can be conveniently analyzed as if they affect only the mass of firms and the average behavior of the firms, despite the fact that emissions trading may influence different firms differently.

2. IMPACT OF AUCTIONED EMISSIONS TRADING

We shall start by analyzing the effects of auctioned emissions trading, relative to no regulation, so as to distinguish the effects of emissions trading from those of particu-
lar allocation schemes. One way to do so may be to consider an economy with \( \tau = 0 \) as a no regulation benchmark. This can be done by reformulating the firm’s cost function and aggregate accounting conditions discussed in the preceding section. Such an analysis would be suited for an economy undergoing a massive change in the regulatory environment. Most economies today, however, have some background regulation, which would force firms to face some positive price of emissions. For example, many countries charge very high taxes on fossil fuels. Introduction of auctioned ET into such economies would mean a further increase in the price of pollution. Given this, we examine the impacts of auctioned ET as those of an increase in the cost of emissions \( \tau \) (or equivalently, a decrease in \( Z \)) on the stationary equilibrium of the economy described in section 1. It turns out all the main results of this section are intact regardless of whether we analyze it as an increase in \( \tau > 0 \) or as a move from \( \tau = 0 \) to some positive price \( \tau > 0 \). Yet, the former makes our exposition substantially simpler.

To start, note that the entry-exit condition (13) does not depend on \( \tau \) or \( Z \). Hence, the cutoff productivity and the size distribution of firms stay the same between no regulation and with auctioned ET: \( \phi_N^* = \phi_A^* \), where subscripts \( N \) and \( A \) refer to no regulation and auctioned ET, respectively. The result may appear somewhat counterintuitive, as it implies that a naive conjecture—that an increased price of emissions may induce exit of less productive (and more pollution-intensive) firms—fails here. However, the result indeed closely parallels that of Spulber (1985) and of Baumol (1988), which state that both (auctioned) emissions trading and emissions tax induce efficient entry/exit in the long run. Indeed, the economic mechanism that underlies Spulber’s study is the same as in ours. The key to understanding the result is to see that the firm’s profit can be completely written as the sole function of the firm’s revenue as in (8). Because all firms face increased marginal cost of production due to the price of pollution and can readjust their markup prices in proportion to their productivity levels, the increase in the permit price affects all firms the same way, including those entrants who decided to exit the market. Hence, the auctioned emissions trading does not favor any particular firm, either productive or unproductive, and thus does not have any impact on industry-wide allocation of firm-level variables. As a result, exactly the same type of firms stay in the market, each with a higher price, a lower output quantity, and a lower labor input level due to increased marginal cost. Section 3 explains more fully that the factor intensity in entry is the real driver in determining whether or not an increased permit price would induce the exit of less productive firms.

That the auctioned ET does not alter the entry-exit condition, however, does not mean that overall economic activity is unaffected. Indeed, an increased cost of emissions does result in a smaller mass of firms and less entry in equilibrium, which again mirrors the result of Spulber and Baumol. To see this, observe from equation (ZCP) that an increase in \( \tau \) raises \( \pi_A \). That is, the average firm profit is higher with
the auctioned ET than without it. This occurs precisely because firms now need to be more profitable to make up for the higher cost of pollution to stay active in the market. Furthermore, because both the average profit and the cost of emissions are higher with the auctioned ET than with no regulation, we see from (18) that the equilibrium mass of firms is smaller, \( M_N > M_A \), and new entry is also smaller, \( N_N > N_A \) (since \( N = \delta M/(1 - G(\phi)) \)).

**Proposition 1**: An auctioned emissions trading (and its associated increase in the price of emissions) does not alter the entry-exit condition, yet reduces the mass of firms and new entry. That is,

\[
\phi^*_A = \phi^*_N, \quad M_A < M_N, \quad N_A < N_N.
\]

Furthermore, the average firm profit is higher under the auctioned ET than under no regulation (i.e., \( \pi_A > \pi_N \)) and is decreasing in the emissions cap \( Z \).

3. IMPACT OF CLOSURE PROVISION

We now examine the impacts of grandfathering under a closure provision, relative to auctioning. Under the closure provision, incumbent firms are allocated some amount of permits freely, yet they lose the permits on a certain condition. The condition is usually the firm’s exit, as with the case of the European Union Emission Trading System (Ellerman and Buchner 2007). The system is in sharp contrast with the US Acid Rain Program, which makes the initial distribution of permits permanent—that is, firms that receive permits retain the permits upon exit, whereas active firms that do not receive permits need to buy permits every period from other firms that hold them. Under the permanent allocation rule, the independence property holds because initial permits are distributed unconditional on all relevant economic decisions, including entry and exit. Hence, the stationary equilibrium outcome of emissions trading would be the same between auction and permanent permit allocation. In appendix B, available online, we offer a proof for this intuitive result.

With the closure provision, however, the initial assignment of permits upon entry serves as an entry subsidy, whereas the loss of permits upon exit serves as an exit tax. We shall demonstrate that this de facto subsidy-tax scheme on entry and exit causes two types of effects, and as a result, at least one qualification for the independence property would fail. However, we shall also see that it is still possible for the regulatory authority to devise an allocation rule so that at least some of the allocative outcomes, such as the cutoff productivity and the permit price, would remain intact.

To demonstrate these points, let us consider a generic allocation scheme in which firms receive permits in proportion to a baseline “business-as-usual (BAU)” emissions level under no regulation. In practice, such a baseline could be a historical average or an industry average. All we require is that the baseline needs to be predetermined so
that it is exogenous to all relevant decisions such as production and abatement. Furthermore, we assume that firms receive these permits only when they are in operation, and the closure provision requires firms to forgo permits when they cease operation. More specifically in terms of our model, this allocation rule implies:

\[ z'_c(\phi) = \begin{cases} \frac{\phi}{\phi_N} \chi^{-1} z_b & \text{if produce}, \\ 0 & \text{otherwise} \end{cases} \]

where \( z_b \) is the baseline emissions level and \( \chi > 0 \). Recall that firms' variable emissions increase at the rate \( \sigma - 1 \) in proportion to productivity \( \phi \). Hence, this formula means that more productive firms would be allocated disproportionately more permits if \( \chi > \sigma \) (i.e., allocation of permits is regressive), whereas less productive firms would be allocated more permits if \( \chi < \sigma \) (i.e., allocation of permits is progressive). We emphasize, however, that our objective here is not to evaluate the effects of a particular allocation rule. Rather, we consider this generic scheme to demonstrate the distortionary effects of any conditional allocation rules that have such a closure provision.

The first impact of the closure provision appears in firms’ entry-exit behavior. Because firms must forgo permits upon exit, the exit condition under this scheme is

\[ \pi(\phi^*_c) = \frac{r(\phi^*_c)}{\sigma} - f^* + \tau z'(\phi^*_c) = 0, \]

so that the cutoff revenue \( r(\phi^*_c) \) is equal to \( \sigma(f^* - \tau z'(\phi^*_c)) \). Rewriting firm’s profit using this, we can rework the equation that governs the entry-exit behavior under the closure provision: 10

\[ (1 - G(\phi^*_c)) \left[ \left( \frac{\phi}{\phi^*_c} \right)^{\sigma - 1} - 1 \right] - \tilde{s}(\chi, \sigma) = \frac{\delta f}{f}, \]

where

\[ \tilde{s}(\chi, \sigma) \equiv \int \tau z_b \left( \frac{\phi}{\phi_N} \right)^{\chi^{-1}} \left[ \left( \frac{\phi}{\phi^*_c} \right)^{\sigma - \chi} - 1 \right] \mu(\phi) d\phi. \]

Comparing (22) with (13) in the case of auctioning, we see that the condition only differs by the second term in the bracket. Note that for all active firms, \( \phi \geq \phi^*_c \) by assumption. Hence, \( \tilde{s}(\chi, \sigma) \geq 0 \) if and only if \( \chi \leq \sigma \). Because the left-hand side without the term \( \tilde{s} \) is decreasing in \( \phi^*_c \) (see Melitz for its proof), we see that \( \phi^*_c(\chi \leq \sigma) < \)

---

9. In this sense, this allocation scheme should be considered a production/closure provision rather than an entry/closure provision. While it may be of some interest to investigate the difference between the two, we leave that out to avoid undue complexity.

10. See appendix A for a detailed derivation of this condition.
$\hat{\phi}_{CP(\chi = \sigma)} = \hat{\phi}_{PA} = \hat{\phi}_{A} = \hat{\phi}_{CP(\chi > \sigma)}$. That is, a regressive allocation would raise the average productivity of firms relative to auctioning while a progressive allocation would lower it.

The key to understanding this point is to recognize that firms receive implicit subsidies (taxes) if firms are distributed permits in such a way that enables them to be net sellers (buyers) of the permits in equilibrium. A progressive allocation would allow less (more) productive firms to be net sellers (buyers), whereas a regressive allocation would allow more (less) productive firms to be net sellers (buyers). For example, if permits are distributed based on a uniform emissions rate and the BAU output level, then given that firms’ emissions rates are decreasing in productivity, such an allocation rule would be regressive and induce exit of less productive firms. In contrast, if firms are allocated permits in proportion to their emissions, then the entry-exit condition is unaffected because such an allocation rule would result in the neutral distribution of permits and favor no particular firm at the intensive margin. This result mirrors that of Böhringer and Lange (2005b, 2044), who find that allocating permits proportionally to past emissions allows firms to “face the same marginal benefits from emissions . . . in subsequent periods.”

We now turn to another impact of the closure provision. To do so, we derive the aggregate accounting equations that would hold for any value of $\chi$. One important distinction between the permanent and nonpermanent allocation rules is that the value of freely distributed permits stays within the market (i.e., it remains with firms that operate in the market), because permits are given only to firms that stay active while the firms that exit must forgo them. In terms of aggregate accounting conditions, this means that the sum of aggregate payments to labor and emissions used in production must equal the difference between the aggregate revenue (in this case, from sales of commodities as well as permits) and the aggregate profit: $L_p + \tau Z_p = R + \tau Z - \Pi$. That is, the incumbent firms receive the permits for free, which they can sell in the emissions market. Using this and following the same steps as under auctioning, we obtain:

$$R = L.$$  \hspace{1cm} (23)

Comparing this with (14), we see that the aggregate revenue is lower under the closure provision than under the auctioned ET given labor endowment $L$. Consider

11. Such a rate-based allocation rule was used in the US SO$_2$ Allowance Program, where each regulated unit received allowances roughly based on the fixed emissions rate (i.e., 2.5 lbs/mmBtu in Phase I and 1.2 lbs/mmBtu in Phase II) and its historical fuel use (which has a roughly one-to-one relationship to its electricity output), with some unit-specific bonus reserves. The allocation is permanent under the US SO$_2$ Allowance Program. Instead, we are discussing the rate-based allocation in the context of entry/closure provision.
next the accounting equation for aggregate emissions (15). Unlike auctioning or permanent allocation, the cutoff profit (21) implies that the average revenue now has a term on the (average) value of permits: \( \bar{\tau} = \sigma(\bar{\pi} + f\tau^A - \tau) \) instead of \( \bar{\tau} = \sigma(\bar{\pi} + f\tau^A) \). Substituting this along with \( M = R/\bar{\tau} \) into the accounting equation (15), we obtain the economy-wide demand for permits:

\[
\tau Z = \frac{\beta R}{1 - \beta}.
\]

Therefore, the closure provision may alter not only the entry-exit condition but also the aggregate resource constraints.

Interestingly, though, the market forces can completely absorb all these distortions, at least in the determination of the permit price. To see this, substitute (23) in (24) and solve for \( \tau \). We then see that the price of permits under the closure provision is identical to that under auctioning in (17). Hence, the price of permits remains the same. Intuitively, grandfathering would endow firms with transferable property rights, which raises the demand for permits relative to auctioning for a given economy-wide income \( R \) (see [16] and [24]). With auctioning, on the other hand, the payments go to the government and eventually to the demand side, which raises the demand for permits relative to grandfathering (see [14] and [23]). In equilibrium, these two competing effects adjust perfectly to exactly offset each other, and hence, the price of permits is still unaffected. Because this result is independent of any particular allocation rules, the invariance of the permit price with respect to the initial distribution of permits still holds even with the closure provision. (To be more precise, though, this permit-price invariance must be understood in terms of relative input prices. That is, \( \tau_A/\omega_A = \tau_{CP}/\omega_{CP} \), where wages are normalized to 1 in this economy.)

Despite this, allocative outcomes would still be different because the initial distribution matters for both the size distribution and mass of firms. Note that equation (24) implies \( \tau z^A = \beta L/[(1 - \beta)M] \). Applying this in \( M = R/\bar{\tau} \), we obtain:

\[
M_{CP} = \frac{(1 - \beta + \sigma\beta)L}{\sigma(1 - \beta)(\bar{\pi}_{CP} + f\tau^A)}.
\]

In this expression, we see that there are two distortionary impacts of the closure provision on the mass of firms. First, there is a pure impact of the closure provision via its effect on aggregate resource constraints. That is, even when the regulator allocates permits in a neutral manner (i.e., \( \chi = \sigma \), in which case \( \phi^*_{CP} = \phi^*_A \) and \( \bar{\pi}_{CP} = \bar{\pi}_A \)), we still have:

\[
M_A/M_{CP} = \frac{1}{1 - \beta + \sigma\beta} < 1,
\]
where the last inequality follows because \(1 - \beta + \sigma \beta = 1 + (\sigma - 1)\beta > 1\) (recall \(\sigma > 1\) by assumption). Second, there is a distortionary effect via its effect on the entry-exit condition. As we discussed above, the cutoff productivity and the average profit are increasing in \(\chi\), so that \(M_{CP}\) is decreasing in \(\chi\). In other words, allocation rules that would induce the exit of less productive firms would support a smaller overall mass of firms.

**Proposition 2:** Suppose that given the cap on aggregate emissions \(Z\), the regulatory authority allocates permits freely with a closure provision. Then neither the size distribution of firms nor the mass of firms is independent of the initial distribution of permits, yet the equilibrium price of permits still remains the same as under auctioning regardless of the initial distribution of permits.

Some may argue that our main claim—the entry/closure provision can alter the size distribution and mass of firms yet still does not distort the price of permits—may depend substantially on the assumption of the same emissions intensities in production and entry. Indeed, factor intensity differences between production and entry can significantly change our result. Intuitively, factor intensity differences may occur for two reasons. First, economic resources required for production may be different from those for entry (e.g., research and development). It is then plausible that firms may face different technologies between production and entry. Second, even if firms’ technical factor intensities are the same, it may not be feasible, either economically or politically, for the regulatory authority to require entering firms to pay the full cost of emissions in entry prior to their operation. If firms generate emissions in entry yet their emissions are not part of the regulatory coverage, then such a regulatory treatment alone may work as a de facto subsidy on entry, the effects of which can vary over different firms with different productivity levels. An important question then is, What kind of distortionary impacts would the factor intensity differences cause on top of those that originate purely from the design of emissions trading? In particular, is our independence result robust to different emissions intensities in production and entry?

To investigate these questions, we let \(\beta_e\) be the emissions intensity in the fixed entry cost and assume \(\beta_e \neq \beta\) in general. We shall focus on auctioned ET and grandfathered ET under the closure provision with \(\chi = \sigma\). Consider first the entry-exit condition. Because all the arguments aside from the fixed entry cost are still intact, the expression for firm’s profit and the resulting zero-cutoff condition remain the same as before. In the free-entry condition, however, we have \(f_\tau \beta\) in place of \(f_\tau \beta\). Thus, substituting the zero cutoff condition into the free-entry condition, we obtain the equation that defines the entry-exit condition:

\[
(1 - G(\phi^*)) \left[ \left( \frac{\phi}{\phi^*} \right)^{-1} - 1 \right] \frac{f_\tau \beta}{\delta} = f_\tau \beta. \tag{26}
\]
Unlike (13) or (22) (with $s = 0$ for $\chi = \sigma$), the cutoff productivity now depends on the price of permits $\tau$. The expected value of entry on the left-hand side and the cost of entry on the right-hand side depend on the factor intensities of production and entry, respectively. These factor intensities, together with the price of emissions (relative to wage), determine whether the cutoff productivity increases relative to the case where $\beta = \beta_e$. For example, suppose $\beta > \beta_e$, so that $\tau^b > \tau^h$ with $\tau/w > 1$. In this case, production would be relatively more costly than entry for a given price of permits. Hence, an increase in the permit price would increase the expected value of entry more than the cost of entry (given $\phi'$). This tends to raise the cutoff productivity level $\phi'$. A key here is to recognize that firm’s productivity is associated only with the (marginal) cost of production. When entry is less pollution intensive than production, an increase in the permit price would make production relatively more costly than entry, thereby inducing the exit of less productive firms. The reverse holds when entry is more pollution intensive than production, in which case the permit price increase would allow low-productivity firms to stay active upon entry.

Now, let us consider the aggregate accounting conditions. Observe first that virtually all the aggregate accounting conditions remain the same as before, so that $R = L + \tau Z$ under auctioning and $R = L$ under grandfathering with closure provision. An exception occurs on the accounting equation for the aggregate emissions. Equation (15) now becomes:

$$\tau Z = \rho \beta R + \beta \frac{R}{\tau} f \tau^h + \beta_e \frac{R}{\tau} \bar{\pi}.$$  \hspace{1cm} (27)

In the case of auctioning, $\bar{\tau} = \sigma (\bar{\pi} + f \tau^h) = \sigma f \tau^h (\tilde{\phi}/\phi')^{r-1}$. Substituting this into (27) and manipulating, we have:

$$\tau Z = (\beta + \kappa (\phi')) R,$$  \hspace{1cm} (28)

where $\kappa (\phi') \equiv (1 - \rho) (\beta_e - \beta) \{ 1 - (\tilde{\phi}/\phi')^{\tau_e - \tau_h} \} < 0$ if and only if $\beta_e < \beta$. We emphasize here that comparing (28) with (16), we immediately see another distortionary impact of the factor intensity difference that does not originate from the design of emissions trading. That is, for a given level of aggregate incomes $R$ and the factor intensity in production $\beta$, the demand for permits is lower if $\beta_e < \beta$ than if $\beta_e = \beta$. The economic intuition is simple enough. The demand for permits is lower than otherwise if firms pollute less in entry or if the regulator does not require firms to hold permits for it. Given the permit endowment $Z$, this decrease in demand for permits results in a lower equilibrium price of permits. To see this, apply $R = L + \tau Z$ and solve for $\tau$. We then obtain the price of permits under auctioning given the emissions cap $Z$:

$$\tau_A = \frac{(\beta + \kappa (\phi')_L)}{(1 - \beta - \kappa (\phi')_L)} Z.$$  \hspace{1cm} (29)
Now let us see how the closure provision adds another layer of distortion. Under the closure provision (with $\chi = \sigma$), we have $r = \sigma(\pi + f\tau' - \tau')^{(\phi')^{-1}}$. Substituting this into (27) and manipulating, we have

$$\tau(1 - \beta_e)Z = (\beta + \eta(\phi'))R,$$

where $\eta(\phi') \equiv (1 - \rho)(\beta_e - \beta)(1 - (\phi'/\phi')^{1-\sigma}(f\tau'/(f\tau' - \tau')^{1}/C30/C3)} < 0$ if $\beta_e < \beta$.

Using $R = L$ and solving for $\tau$, we obtain the price of permits under the closure provision given the emissions cap $Z$:

$$\tau_{CP} = \frac{(\beta + \eta(\phi'_{CP}))L}{(1 - \beta_e)Z}.$$

Comparing (29) and (30), we see that $\tau_{CP} \neq \tau_A$ and $\phi'_{CP} \neq \phi'_A$ in general.

**Proposition 3**: If firms face different factor intensities in production and entry, then neither the size distribution of firms nor the price of permits would be the same under the auctioned and the grandfathered emissions trading with the closure provision.

4. IMPACT OF OUTPUT-BASED ALLOCATION RULE

With an output-based allocation rule, all new entrants are allocated some amount of permits freely as a rebate to their production. Firms forgo the permits upon exit by definition because they do not produce after exit. Such a rule was proposed in the Waxman-Markey legislation, and several variations of it were investigated in the previous studies. For example, Fischer and Fox (2007) considered allocations based on firms’ output shares within each sector, whereas Fowlie et al. (forthcoming) considered allocations based on an industry-specific emissions rate from a previous period. In this paper, we interpret the OBA as an allocation scheme based on firms’ output shares in the industry. Such an OBA rule serves as a de facto rebate not only on firms’ production status but also on production amounts. Hence, the OBA rule may alter firms’ incentives directly not only at the extensive margin (i.e., entry/shutdown decisions) but also at the intensive margin (i.e., production/emissions decisions).

Formally,

$$z_{OBA}(\phi) = \begin{cases} 
\frac{q_{OBA}(\phi)\rho}{Q_{OBA}}Z & \text{if produce} \\
0 & \text{otherwise}
\end{cases}.$$  

(31)

Note that in this formula, output shares are unit-adjusted to account for the fact that products are differentiated across firms. Recall that in our setup (as in Melitz [2003] and other subsequent studies), $Q$ is an aggregate output index defined as $Q = \left[\int q(\omega)\rho d\omega\right]^{1/\rho}$. Hence, this adjustment is necessary to ensure that these output shares integrate to 1. Put differently, the rebate rate in our formulation is based on the value-added measure of output and is indeed consistent with the previous studies.
that investigate the impacts of the OBA rule in an economy consisting of multiple sectors (e.g., Böhringer and Lange 2005a; Fischer and Fox 2007). To see this point, consider (as in these previous studies) the ad valorem subsidy per unit of the value-added measure of output:

$$s(\phi) = \frac{\tau}{\int p(\phi)q(\phi)M\mu(\phi)d\phi}Z.$$ 

Because \( p = Q^{1/\alpha}Pq^{-1/\alpha} \), the amount of production rebate each firm receives is given by

$$s(\phi)p(\phi)q(\phi) = \frac{p(\phi)q(\phi)}{\int p(\phi)q(\phi)M\mu(\phi)d\phi}Z = \frac{\tau Q^{1/\alpha}Pq^0}{PQ}Z = \frac{q^0}{Q^0}Z = \tau z'(\phi).$$

Hence, our formulation of the OBA rule coincides with the conventional OBA rule used in empirical applications.12

Before starting our analysis, we clarify one important assumption concerning how permit allocation is treated in our model. Given the emissions cap \( Z \), the amount of permits each firm receives depends on its share in the aggregate output that would arise in the equilibrium under this allocation rule. Moreover, the emissions cap must equal the total amount of permits allocated for firms that enter and stay active in equilibrium. To ensure this, we assume that firms have perfect foresight about all aggregate economic variables, so that they can perfectly anticipate their own permit allocations \( z' \) prior to entry given the knowledge of \( Z \) and the allocation rule. By this, we are implicitly assuming that firms only anticipate how many permits they would receive upon entering the market, and that firms do not expect either their entry/exit or their output/emissions to influence the distribution of permits. This behavioral assumption is employed in the study of the impact of permit allocation rules by Fowlie et al. (forthcoming), and is also consistent with virtually all economic analyses of perfectly competitive markets concerning the equilibrium prices.

Under the OBA rule, the firm with productivity \( \phi \) under the endogenous OBA solves

$$\max q'p(q) - c(q) + \tau q^0Z, \quad (32)$$

taking \((\tau, Q, Z)\) as given. Note that the firm's cost function is still the same as in (4) because the firm's cost-minimizing choice of factor inputs remain the same given \( q \). The last term in (32) is the initial receipt of permits, the effect of which the firm now internalizes by adjusting its output and price markup. Intuitively, this

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12. Due to monopolistic competition, the output-based rebate to each firm is no longer linear in its own output as in the case of perfect competition. The OBA rule applied to oligopolistic firms (with market power in the output market) in Fischer (2011) has the same nonlinear feature.
pervasive incentive to increase output should increase demand for permits and thus the price of permits relative to the case of auctioning for a given emissions cap, as in Fisher and Fox (2007). Furthermore, because the OBA is also a de facto subsidy on entry, the pervasive incentive on the extensive margin should also increase the equilibrium mass of firms relative to auctioning. It turns out all these economic intuitions indeed hold true in our model.

The remaining question then is, Would this de facto rebate on production also distort firm’s entry-exit condition, thereby distorting the size distribution of firms? The answer turns out to be no. To see this point, let us solve (32). We then see that firms adjust their output and price markups in proportion to their productivity levels:

\[
p(\phi) = \left(\frac{p\phi}{\tau^0}\right)^{-1}, \quad q(\phi) = Q \left(\frac{Pp\phi}{\tau^0}\right)^{\frac{a}{\beta}}, \quad z_{pR}(\phi) = \frac{p\phi}{\tau^0} \left(\frac{Pp\phi}{\tau^0}\right)^{\frac{a-1}{\beta}},
\]

where \(\gamma \equiv 1 + \tau(Z/R)\). It thus follows that because firms are distributed permits in proportion to its output share (and conditioned on production rather than entry) and because each firm’s output is proportional to its productivity, the OBA rule results in the neutral distribution of permits and favors no particular firm. While we leave the formal proof in appendix A, we can see this effect directly by observing that plugging \(q(\phi)\) from (33) into (31) yields \(z'_{OBA}(\phi) \propto \phi^{a-1}\). Hence, the proportional response in output leaves the cutoff productivity unchanged, much like the same way as the neutral allocation rule \((\chi = \sigma)\) under the closure provision. Put differently, the OBA rule serves as implicit subsidies on firms’ exogenous productivity levels. An important point here, though, is that unlike (20), this occurs indirectly through firms’ endogenous choice of output and markups.

In sum, though the OBA rule is indeed a conditional allocation rule, it would not affect the entry-exit condition, yet it would distort the aggregate resource constraints via its effect on firms’ output decisions. Consequently, it results in a violation of the independence property in terms of the price of permits. These results are in sharp contrast to the case of the closure provision.

**Proposition 4:** Suppose that given the cap on aggregate emissions \(Z\), the regulatory authority allocates permits freely in proportion to firms’ output shares. Then the size distribution of firms stays the same as under auctioning. Yet the equilibrium price of permits, the mass of firms, and the new entry would be higher than under auctioning.

5. WELFARE IMPLICATIONS

The main result of the previous sections—that design of emissions trading can affect size distribution, mass of firms, and permit price by altering the entry-exit and aggregate accounting conditions—has substantial implications for social welfare. In
real-world settings, either the allocation scheme or the emissions cap or both are chosen primarily through political processes and, thus, are often outside the regulatory agency’s control. We shall see that given the emissions cap, social welfare can vary substantially across different allocation schemes, showing important interactions with size distribution, mass of firms, and permit price. To demonstrate this point, we shall examine social welfare under three allocation schemes: auctioning, grandfathering with closure provision, and grandfathering with the OBA.

As in Melitz, consider per capita utility of the aggregate consumer as the measure of social welfare: \( W \equiv U/L = Q/L - h(Z) \). Observe that given the allowable economic resources \( L \) and \( Z \), social welfare depends entirely on the size of the aggregate output index \( Q \). Because \( Q = R/P \) and \( P = M^{1/(1-\sigma)} p(\hat{\phi}) \), applying the markup pricing rule under each scheme, we see that the relative impacts of allocation schemes can be decomposed as follows: For any allocation schemes \( i \) and \( j \),

\[
\frac{Q_i}{Q_j} = \left( \frac{R}{R} \right)_{\text{Agg Income}} \times \left( \frac{M}{M} \right)_{\text{Mass/Variety}} \times \left( \frac{\phi}{\phi} \right)_{\text{Avg Productivity}} \times \left( \frac{mu}{mu} \right)_{\text{Markup Factor}} \times \left( \frac{\tau}{\tau} \right)_{\text{Factor Price}},
\]

where the markup factor \( mu \) is \( 1/\rho \) for auctioning, permanent allocation, and closure provision, and \( 1/\rho \tau = \{1 - \beta(1+\rho)\} / [\rho(1 - \rho \beta)] \) for the OBA.

This equation indicates that the overall impact on aggregate output index is composed of five competing effects on economy-wide income, mass of firms (or equivalently, product variety), weighted average productivity (or equivalently, size distribution), markup, and relative factor prices (in this case, permit price only as \( w \) is normalized to 1, so it is represented only by \( \tau \)). Higher aggregate income, mass of firms, and average productivity all tend to increase aggregate output, whereas higher markup factor and factor price tend to decrease it. Therefore, all of the identified intra-industry effects of allocation schemes discussed in previous sections will interact with one another in determining the aggregate output index and the social welfare. The question then is, Which of the effects tends to dominate in each allocation scheme?

Let us first compare auctioning versus closure provision. Recall that the permit price stays the same between the two schemes and that the weighted average productivity \( \hat{\phi} \) critically depends on the regressiveness of allocation rule \( (\chi) \) under the closure provision. To avoid undue complexity, let us consider the case of neutral allocation \( (\chi = \sigma) \), so we have \( \hat{\phi}_A = \hat{\phi}_{CP} \). Then the last three terms of (34) cancel out. As discussed in section 3, the aggregate income is higher under auctioning than under closure provision by a factor \( (1 - \beta) \), which tends to favor auctioning, whereas the mass of firms is lower under auctioning than under closure provision, which tends to favor closure provision. It turns out that the former effect dominates the latter. Specifically we have:

\[
\nu_{CP} \equiv \frac{Q_{CP}}{Q_A} = (1 - \beta)(1 - \beta + \sigma \beta)^{1/(1-\sigma)} \leq 1,
\]
where the last inequality follows because the ratio equals 1 when \( \beta = 0 \) and its derivative is negative. We defer a detailed proof of all of the results in this section to appendix A. Hence, the aggregate output (and the welfare) is higher under auctioned ET than under grandfathered ET with closure provision for a given \( Z \).

How about the OBA? As with the closure provision, the aggregate income is higher under auctioning than under the OBA, the mass of firms is lower under auctioning than under the OBA, and the weighted average productivity is the same between the two schemes. In this case, however, the OBA also induces a higher price of permits than auctioning, which tends to favor auctioning, whereas the markup is smaller under the OBA than auctioning, which tends to favor the OBA. Thus, the aggregate output would be lower under the OBA than auctioned ET, unless the OBA induces a substantially larger mass of firms or a substantially smaller markup than auctioning. It turns out that it does:

\[
\nu_{\text{OBA}} \equiv \frac{Q_{\text{OBA}}}{Q_A} = (1 - \rho \beta) \left[ \frac{1 - \beta (1 + \rho)}{1 - \beta} \right]^{(\beta - 1)/\rho} \left[ \frac{1 - \rho + \rho^2 \beta}{1 - \rho} \right]^{1/(\sigma - 1)} \geq 1,
\]

where the last inequality follows because the ratio equals 1 when \( \beta = 0 \) and its derivative is positive.

Note that the ratios of aggregate output indices do not depend on the size of the permit endowment \( Z \). Examination of the output-decomposition equation (34) explains why. The aggregate income and the markup factor are both independent of \( Z \) under any emissions trading schemes. The ratio of \( \tau \)'s is also independent of \( Z \). We have shown that \( \phi_A^* \) and \( \phi_{\text{OBA}}^* \) are independent of \( Z \) while the proof of proposition 2 demonstrates that \( \phi_{\text{CP}}^* \) is independent of \( Z \) as well (regardless of whether \( \chi = \sigma \) or not). The mass of firms, \( M \), does depend on \( Z \). However, it can be shown that \( M \) under each regime is linear in \( \tau^{-\beta} \), and thus, the ratio of \( M \)'s is independent of \( Z \). Hence, the ratios \( Q_i/Q_j \) are independent of the size of the aggregate emissions cap for any two emissions trading schemes \( i, j \).

13. To see this, observe that the expression in (18) can be written as

\[
M_A = \frac{L}{\sigma(1 - \beta)(\phi_A^*/\phi_A^*)^{\sigma - 1}} \int \tau^{\rho} d\tau,
\]

whereas in (A1) and (A2) in appendix A, we demonstrate that

\[
M_{\text{CP}} = \frac{(1 - \beta + \sigma \beta)L}{\sigma(1 - \beta)(1 - \rho + \rho \beta)(\phi_{\text{CP}}^*/\phi_{\text{CP}}^*)^{\sigma - 1}} \int \tau^{\rho} d\tau,
\]

\[
M_{\text{OBA}} = \frac{(1 - \beta (1 + \rho - \sigma))L}{\sigma(1 - \beta (1 + \rho))(\phi_{\text{OBA}}^*/\phi_{\text{OBA}}^*)^{\sigma - 1}} \int \tau^{\rho} d\tau,
\]

where \( \Phi_1 \) and \( \Phi_2 \) are defined in A1 and independent of \( Z \).
The discussion so far establishes that $Q_{CP} \leq Q_A \leq Q_{OBA}$; that is, the OBA scheme induces the highest aggregate output (and social welfare) among the three allocation schemes given the emissions cap. The result is consistent with Fowlie et al. (forthcoming) as well as the rationale behind the Waxman-Markey legislation that the OBA is a viable means to compensate for the increased cost of pollution control.

A flip side of this result is that the emissions cap could be optimally adjusted to improve social welfare in second-best settings where choice over allocation schemes is politically constrained. A natural question then is, How should the regulator adjust the cap in such settings? One may be tempted to conjecture that the emissions cap should be increased (decreased) for the closure provision (the OBA) because the output is lower (higher) then. Such an argument is correct, however, only if the regulatory authority is politically constrained to achieve a given level of social welfare. Indeed, if the regulator is free to choose the cap, it should lower it under the closure provision and raise it under the OBA relative to the auctioned ET.

To see this point, consider the second-best planner’s problem, in which the regulatory authority maximizes social welfare but is allowed to choose only $Z$, subject to a predetermined allocation rule. Substituting relevant expressions for $R$, $M$, and $\tau$ as before and manipulating, we can rewrite the social welfare under each scheme:

$$W = \begin{cases} \Gamma(\phi_A^*)Z^{h/\rho} - h(Z) & \text{under auctioning} \\ \nu_{CP}\Gamma(\phi_{CP}^*)Z^{h/\rho} - h(Z) & \text{under the closure provision,} \\ \nu_{OBA}\Gamma(\phi_{OBA}^*)Z^{h/\rho} - h(Z) & \text{under the OBA} \end{cases}$$

where

$$\Gamma(\phi^*) \equiv (1 - \beta)^{-1} \rho \phi^*[L / \{\sigma(1 - \beta)f\}^{1/(\sigma - 1)}} \{1 - \beta\} / (\beta L)^{\beta/\rho}. \tag{35}$$

Note that in general, the optimal emissions cap should also depend on the size distribution because $\Gamma$ depends on $\phi^*$. However, $\phi^*$ does not depend on $Z$ (regardless of $\chi = \sigma$ or not) as shown in appendix A, (A1). In addition, $\nu$’s are constants given the exogenous parameters of the model. Hence, we can easily examine the impact of varying $Z$ on social welfare, assuming that $W$ is concave in $Z$. Observe, first, that the marginal social cost of increasing the cap (i.e., $h'(Z)$) is the same under all schemes. Second, the marginal social benefit of an additional unit of permits is higher (lower) under the OBA (the closure provision) than auctioning:

$$(\beta/\rho)\nu_{CP}\Gamma(\phi_{CP}^*)Z_{CP}^{(h/\rho) - 1} < (\beta/\rho)\Gamma(\phi_A^*)Z_{A}^{(h/\rho) - 1} < (\beta/\rho)\nu_{OBA}\Gamma(\phi_{OBA}^*)Z_{OBA}^{(h/\rho) - 1},$$

precisely due to its impact on aggregate output. Therefore, as long as the marginal social benefit is decreasing and the marginal social cost is nondecreasing, the second-best optimum satisfies $Z_{CP} \leq Z_A \leq Z_{OBA}$. That is, the regulator should raise the
cap for the OBA scheme relative to the auctioned scheme (and for the auctioned scheme relative to the closure provision) precisely due to the difference in the marginal social benefits of emissions under the three schemes.

Of course, $\beta/\rho$ may or may not be less than 1 because $\beta$ and $\rho \in (0, 1)$. Hence, we would need $h$ to be sufficiently convex for $W$ to be concave in $Z$. Or alternatively, we could impose curvature on the utility from $Q$ and assume a constant marginal damage on $h$. For example, we could instead assume $W = \log(Q/L) - hZ$ as a special case. In such a case, the impact of $h$ on the social benefit is additively separable from that of $Z$, so that we would have $Z_{CP} = Z_A = Z_{OBA}$.

**Proposition 5**: (i) Given emissions cap $Z$, the social welfare and aggregate output index differ substantially across allocation schemes, due to changes in the entry-exit and aggregate accounting conditions: that is, $W_{OBA} \geq W_A \geq W_{CP}(\chi = \sigma)$ and $Q_{OBA} \geq Q_A \geq Q_{CP}(\chi = \sigma)$. (ii) Consider the second-best planner’s problem in which choice over allocation schemes is politically constrained. Suppose $h(Z) = Z^a$ and $a$ is sufficiently large (i.e., $a \geq \max \{1, \beta/\rho\}$) to ensure the sufficient condition of the problem. Then the optimal emissions cap satisfies the following relationship: $Z_{CP}(\chi = \sigma) \leq Z_A \leq Z_{OBA}$.

6. NON-POLLUTION-INTENSIVE SECTOR

One important qualification for our main result is the full-employment assumption. We explicitly use this assumption in deriving the mass of firms and the price of permits. Presumably, though, an introduction of emissions trading would cause reallocation of employment from pollution-intensive industries to less pollution-intensive industries. Hence, the full employment assumption would be more appropriate in the model incorporating two or more industries with different pollution intensities. In this section, we shall examine if the price-invariance result still holds in such an economy.

It is sufficient to consider an economy with two sectors in order to convey our main points. Following Copeland and Taylor (1994) and Bernard et al. (2007), let the preferences of the representative consumer be given, in place of (1), by

$$U = \sum_k \alpha^k \ln(Q^k) - Lh(E),$$

where $\sum \alpha^k = 1$ and $Q^k$ is the composite good for industry $k$ defined by

$$Q^k = \left[ \int_{\omega \in \Omega^k} q^k(\omega)^{\rho} d\omega \right]^{1/\rho}.$$

On the producer side, firms in each sector maximize profits with the same production and entry costs (4) and (9) as before, except that firms in different sectors have
different $\beta$'s. Assume sector 1 is more pollution intensive than sector 2: $\beta^{1} > \beta^{2}$. Labor and permits are freely mobile across sectors. We leave all other components of the model unchanged. Under this setup, we shall compare the auctioned ET versus the grandfathered ET with closure provision. For expositional ease, we focus on the case of neutral allocation: $\chi = \sigma$.

Let us first observe that as long as factor intensities are the same in production and entry for each sector, cutoff productivities are the same between auction and grandfathering with closure provision for each sector: $\phi_{Ak}^{c} = \phi_{Ak}^{cp}$. Moreover, virtually all the aggregate accounting conditions remain the same as before, so that $R = L + \tau Z$ under auctioning and $R = L$ under grandfathering with closure provision. Exceptions occur on the accounting equations for factors of production, which now must be balanced for each sector $k$:

$$\tau Z_{k}^{a} = \tau Z_{k}^{pv} + \tau Z_{k}^{pf} + \tau Z_{k}^{e},$$

$$L_{k}^{a} = L_{k}^{pv} + L_{k}^{pf} + L_{k}^{e},$$

The Cobb-Douglas utility function implies that the consumer’s aggregate expenditures on each composite good $R_{k}$ should equal $\alpha_{k} R$. Therefore, all the distortionary impacts of allocative design materialize through their impacts on how the aggregate expenditures on each sector $R_{k}$ are allocated across different factors of production.

Working through (36) and (37), we obtain equilibrium emissions and labor employment in each sector in the case of auctioning:

$$Z_{k}^{a} = \frac{\alpha_{k} \beta^{k}}{\theta} Z$$ and $$L_{k}^{a} = \frac{\alpha_{k} (1 - \beta^{k})}{1 - \theta} L,$$

where $\theta = \alpha^{1} \beta^{1} + \alpha^{2} \beta^{2}$. On the other hand, in the case of grandfathering with closure provision, we have

$$Z_{k}^{cp} = \frac{\alpha_{k} \beta^{k} (1 - \hat{\theta}) + \hat{\alpha}_{k} \beta^{k} \hat{\theta}}{\theta} Z$$ and $$L_{k}^{cp} = \frac{[\alpha_{k} (1 - \beta^{k}) (1 - \hat{\theta}) + \hat{\alpha}_{k} (1 - \beta^{k}) \hat{\theta}] L}{1 - \theta},$$

where $\hat{\alpha}_{k}$ is the share of permits allocated to sector $k$ with $\sum \hat{\alpha}_{k} = 1$ and $\hat{\theta} = \hat{\alpha}^{1} \beta^{1} + \hat{\alpha}^{2} \beta^{2}$.

Implications of the result are substantial. First, in the case of auctioning, labor employment in each sector is determined only via the primitive parameters of the model and labor endowment. Therefore, labor employment in each sector is unaffected by any changes in the price of permits or in the emissions cap. Second, in the case of closure provision, however, labor employment in each sector depends clearly on the share of permits allocated to that sector. Indeed, employment in the pollution-intensive sector is increasing in the share of permits distributed to that sector. Hence, in the absence of other preexisting distortions, the closure provision is the real driver in inducing reallocation of employment from pollution-intensive indus-
tries to less pollution-intensive industries when emissions trading is introduced to an economy. Interestingly, even if the regulator allocates permits in a way that the share of permits to each sector equals the aggregate expenditures for that sector (i.e., \( \hat{\alpha}^k = \alpha^k \)), there would still be active trading across sectors. To see this, plug \( \hat{\alpha}^k = \alpha^k \) in (39) and observe:

\[
Z_{cp}^k - \hat{\alpha}^k Z = \frac{\alpha^k}{\beta^k - \theta} \theta Z,
\]

where \( \beta^1 > \theta \) and \( \beta^2 < \theta \). Hence, the pollution-intensive sector would be the net buyer while the non-pollution-intensive sector would be the net seller of permits.

Let us now examine how these allocative impacts of permit distribution affect the determination of the permit price. Intuitively, given the price-invariance result in section 3, the closure provision per se should not distort the price of permits as long as the authority distributes permits in a way that does not distort the real economic opportunities available to each firm in each sector, for then the competitive permit market should absorb any differences in demand for permits across sectors. In our setup, such a nondistortionary distribution would mean \( \chi = \sigma \) and \( \hat{\alpha}^k = \alpha^k \). Alternatively, if the regulator allocates permits to the pollution-intensive sector more than its aggregate income share (i.e., if \( \hat{\alpha}_1 > \alpha_1 \)), then it should raise the permit price (relative to auctioning) because such an allocation should raise the overall demand for permits. It turns out that these economic intuitions indeed hold. In the case of auctioning, the price of permits equals

\[
\tau_A = \frac{\theta L}{(1-\theta)Z},
\]

whereas in the case of closure provision, we have

\[
\tau_{cp} = \frac{\theta L}{(1-\theta)Z}.
\]

Comparing (40) with (41), the permit price is the same between auctioning and closure provision if and only if \( \hat{\alpha} = \alpha^k \), and \( \tau_A < \tau_{cp} \) if \( \hat{\alpha}^1 > \alpha^1 \) because \( \beta^1 > \beta^2 \) by assumption.

In sum, under the closure provision, initial distribution of permits across sectors has real impacts, not only on the permit price, but also on the inter-industry allocations of emissions and labor. On the one hand, the expenditure share on each sector is determined by consumer preferences and, therefore, is independent of the design of emissions trading. On the other hand, permit distribution that is conditioned on entry/production status may result in perverse incentives for firms’ entry and production. Consequently, distributing permits in a way that favors a pollution-intensive industry can increase the real economic opportunities available in that industry, thereby increasing the industry’s demand for labor and permits and shift-
ing labor and emissions away from a less pollution-intensive industry. This in turn results in an overall increase in the demand for permits and increases the equilibrium permit price.

**Proposition 6:** Consider an economy consisting of two sectors with different pollution intensities. Then the equilibrium price of permits would be the same under the auctioned emissions trading and the grandfathered emissions trading with entry/closure provision if and only if permits are allocated in such a way that the share of permits distributed in each sector equals the expenditure share for that sector. The sectoral emissions and labor employment, as well as the mass size of firms in each industry, all depend on the permit distribution across sectors.

7. **CONCLUDING REMARKS**

This paper examined the long-run impacts of conditional allocation rules under emissions trading in the Melitz-type economy that accounts for endogenous entry/exit of heterogeneous firms. The model allows us to make one important distinction in identifying the allocative impacts, that is, a distinction between the effect on size distribution of firms versus that on the mass of firms. This distinction is important not only because we can clarify the nature of distortion in entry but also because it confers a distinction between the average firm behavior versus the aggregate behavior of the industry. We then considered a suit of allocation schemes in a way to increment policy treatments: from auctioning to grandfathering with permanent allocation, to grandfathering with entry/closure provision, and finally to grandfathering with output-based allocation. The incremental policy treatment, combined with the aforementioned advantage of the model, allowed us to fully disentangle the sources and effects of distortions created through conditional allocation rules.

Our first set of results is that the auctioned ET does not alter the entry-exit condition, and therefore, the cutoff productivity (i.e., the lowest productivity of firms that enter the market in equilibrium) under the auctioned ET stays the same as under no regulation. However, as expected, a smaller mass of firms enter under the auctioned ET with a higher average profit relative to no regulation because firms faced with a positive price of pollution need to be more profitable in order to stay active in the industry.

Second, grandfathering, or free distribution of permits, per se is shown to have no effect on the entry-exit condition. With permanent allocation, firms who receive permits upon entry retain the permits upon exit, whereas firms who did not receive permits need to buy permits every period from other firms who hold them. Because the allocation of permits does not depend on firms’ entry-exit status, such a permanent allocation rule does not distort the entry-exit conditions or aggregate accounting conditions, regardless of how permits are allocated initially. As a result, the auctioned ET and the permanent allocation rule result in the same stationary equilibrium—
despite the fact that the transferable property rights were freely distributed under the grandfathered ET.

Things change dramatically under conditional allocation rules, however. Under the entry/closure provision (as in the EUETS), new entrants are allocated some amount of permits freely while firms lose permits upon exit. Under such a provision, neither the cutoff productivity nor the mass of firms is independent of the initial distribution of permits. We show that the initial distribution of permits may alter the size distribution of firms if permits are distributed in a manner disproportional to firms’ productivity levels (and therefore, their unconstrained emissions levels). If, for example, firms are allocated permits based on a uniform emissions rate (as in the US Acid Rain Program), then the initial distribution of permits would be regressive (i.e., less productive firms would be distributed smaller amounts of permits relative to their unconstrained emissions), so that they would become net buyers of permits, whereas more productive firms would become net sellers. As a result, such a rate-based allocation rule would induce exit of less productive firms and raise the cutoff productivity.

Importantly, however, this distortion in the entry-exit conditions does not necessarily distort the price of permits—it still remains the same as under the auctioned ET. With entry/closure provision, the value of payments by net buyers of permits must equal that of sales by net sellers of permits in the stationary equilibrium. In contrast, with permanent allocation, the payments go to the government and eventually to all the market participants (i.e., consumers/firms), which raises the demand for permits relative to the case of entry/closure provision. However, with entry/closure provision, the free distribution of permits also encourages entry of any firms, which also raises the demand for permits. In equilibrium, these two effects exactly offset each other. Consequently, the price of permits is unaffected. With a two-sector model, however, initial distribution of permits across sectors is shown to have real impacts on the equilibrium price of permits, because it can influence the real demand for permits via inter-industry reallocations of employment, emissions, and firms.

The OBA rule further confounds these effects. Because the OBA rule allocates permits based on firms’ output, it serves as a rebate not only on production but also on entry because firms forgo permits upon exit as they cease their production. A priori then, one would expect the OBA rule to distort both entry/exit and production behavior. It turns out, however, that while the OBA does distort production behavior and the mass and entry of firms, it does not distort the entry-exit behavior. Because firms receive a rebate on the amount of production, all firms face the incentive to increase their supply relative to the auctioned ET. This increases the demand for emissions, and as a result, raises the price of permits compared to the auctioned ET. However, because firms produce outputs according to their productivity levels, the allocation of permits in proportion to output shares constitutes an
allocation of property rights based on their innate productivity levels. Hence, the OBA rule does not, in principle, distort the entry-exit condition.

These impacts of conditional allocations schemes on size distribution, mass of firms, and permit price have real implications for welfare under second-best settings, in which design of allocation rules is politically constrained. We demonstrate that in such a second-best setup, the (constrained) "optimal" emissions cap must be adjusted for specific design features of emissions trading, taking into account their impacts on size distribution, mass of firms, and permit price.

Furthermore, we investigate the implications of two important assumptions of the model: emissions cost in entry and full employment. When factor intensities differ between production and entry, the independence property with respect to the price of permits no longer holds even with the entry/closure provision. In a model with two sectors with different pollution intensities, the aggregate emissions, labor employment as well as mass of firms in each sector all are shown to depend on the initial distribution of permits across sectors. We thus conclude that whether the price-invariance property holds with conditional allocation rules or not depends not only on their specific design features but also on entry cost structures and the coverage of non-pollution-intensive sectors in emissions trading.

These results suggest a new and important pathway for future empirical studies in the growing field of empirical environmental economics. Taken at a different angle, our findings suggest, first, that environmental regulation that supports the same price of pollution may induce different size distributions, mass sizes, and new entries of firms within and across industries under different regulatory design features. Second, an increased cost of pollution may or may not induce exit of more pollution-intensive firms, depending on the technology and regulatory environment that defines the factor intensity of entry cost. And when it does, different designs of emissions trading should generally induce different prices of pollution even for the same emissions cap. The proposed framework offered herein is also widely applicable for assessing other regulatory instruments such as emissions taxes, abatement subsidies, and command-and-control policies. Hence, it would allow us to formulate rich testable hypotheses as to how environmental regulation affects the industry size, entry/exit, and productivity distribution of regulated firms. Further exploration of such a pathway is left for future research.

APPENDIX A
PROOFS OF PROPOSITIONS

A1. Proof of Proposition 2
Here, to complete the proof of proposition 2, we derive the entry-exit condition under closure provision. Using (6) and (21), the firm’s profits can be written as:
\[
\pi_{CP}(\phi) = \left[\left(\frac{\phi}{\phi_{CP}}\right)^{\sigma-1} - 1\right] f \tau^\beta - \tau \left[\left(\frac{\phi}{\phi_{CP}}\right)^{\sigma-1} z'(\phi_{CP}) - z'(\phi)\right].
\]

Applying (20), the second term of the above equation can be rewritten as
\[
\tilde{s}(\phi; \chi, \sigma) \equiv \tau \left[\left(\frac{\phi}{\phi_{CP}}\right)^{\sigma-1} z'(\phi_{CP}) - z'(\phi)\right] = \tau z_b \left[\left(\frac{\phi}{\phi_{N}}\right)^{\chi-1} \left(\left(\frac{\phi}{\phi_{CP}}\right)^{\sigma-\chi} - 1\right)\right].
\]

It then follows that the ZCP condition is given as:
\[
\bar{\pi}_{CP} = \pi(\phi_{CP}^*) = \left[\left(\frac{\phi_{CP}^*}{\phi_{CP}}\right)^{\sigma-1} - 1\right] f \tau^\beta - \tilde{s}(\chi, \sigma),
\]
where \(\tilde{s}(\chi, \sigma)\) is defined as in the main text. Then combining this with the FE condition, we obtain the entry-exit equation (22) as in the main text:
\[
(1 - G(\phi_{CP}^*)) \left[\left(\frac{\phi_{CP}^*}{\phi_{CP}}\right)^{\sigma-1} - 1\right] - \frac{\tilde{s}(\chi, \sigma)}{f \tau^\beta} = \frac{\delta_f}{f}.
\]

It may appear that the cutoff \(\phi_{CP}^*\) depends on \(\tau\) (or the permit endowment \(Z\)) because the second term inside the braces has the term \(f \tau^\beta\). However, this is not true. The key here is that the size of \(z_b\) has to be restricted by the size of \(Z\):
\[
Z \equiv \int z'(\phi) M(\phi) d\phi = z_b M \int \left(\frac{\phi}{\phi_{N}}\right)^{\chi-1} \mu(\phi) d\phi.
\]

Applying this in \(\tilde{s}\) along with (25) and manipulating, we obtain:
\[
\tilde{s} = \frac{\Phi_1}{\Phi_2} \frac{\sigma \beta \left(\frac{\phi}{\phi_{CP}}\right)^{\sigma-1} f \tau^\beta}{1 - \beta + \sigma \beta + \sigma \beta (\Phi_2 / \Phi_1)}.
\]

where
\[
\Phi_1 \equiv \int \left(\frac{\phi}{\phi_{N}}\right)^{\chi-1} \mu(\phi) d\phi, \quad \text{and} \quad \Phi_2 \equiv \int \left(\frac{\phi}{\phi_{N}}\right)^{\chi-1} \left[\left(\frac{\phi}{\phi_{CP}}\right)^{\sigma-\chi} - 1\right] \mu(\phi) d\phi.
\]

Thus the term \(f \tau^\beta\) (and also \(z_b\)) in (22) cancels out. Hence, the cutoff productivity \(\phi_{CP}^*\) does not depend on either \(\tau\) or \(Z\) (regardless of \(\chi\)). The equilibrium mass size is given by
\[
M_{CP} = \frac{(1 - \beta + \sigma \beta) L}{\sigma (1 - \beta) \left(1 - \Phi_1 / \Phi_2 \frac{\sigma \beta}{1 - \beta + \sigma \beta (\Phi_2 / \Phi_1)} \left(\frac{\phi}{\phi_{CP}}\right)^{\sigma-1}\right) f \tau^\beta}.
\]
A2. Proof of Proposition 4

First, to show that the cutoff productivity stays the same, rewrite the firm’s profit as

\[ \pi(\phi) = \left\{ \left( \frac{\phi}{\phi_{OBA}} \right)^{\sigma - 1} - 1 \right\} f \tau^\beta - \tau \left\{ z'(\phi_{OBA}) \left( \frac{\phi}{\phi_{OBA}} \right)^{\sigma - 1} - z'(\phi) \right\}, \]

where the expression inside the second braces cancels out because

\[ z'(\phi_{OBA}) \left( \frac{\phi}{\phi_{OBA}} \right)^{\sigma - 1} - z'(\phi) = \frac{q(\phi_{OBA})}{Q^\gamma} \left( \frac{\phi}{\phi_{OBA}} \right)^{\sigma - 1} - \frac{q(\phi)^{\gamma}}{Q^\gamma} = 0. \]

Hence, the expressions for the zero cutoff profit is the same as (ZCP). Because the FE condition also stays the same, we have \( \phi^*_A = \phi^*_{OBA} \).

Now consider the accounting equation for aggregate emissions:

\[ \tau Z = \tau Z_{pv} + \tau Z_{pf} + \tau Z_e. \]

From individual firms’ optimality conditions, \( \tau z_{pv}(\phi) = \rho \beta \gamma r(\phi) \) and \( \tau z_{pf} = \beta f \tau^\beta \). Integrating them over all firms, we have \( \tau Z_{pv} = \rho \beta \gamma R \) and \( \tau Z_{pf} = \beta M f \tau^\beta \). Moreover, the Cobb-Douglas specification of the entry cost implies \( \tau Z_e = \beta \Pi = \beta M \pi \). Substitute these into the accounting equation above, and apply \( M = R/\tau \) and \( r = \sigma(\pi + f \tau^\beta - \tau^\beta) \). We then obtain

\[ \tau Z = \rho \beta \gamma R + \beta \frac{R}{\sigma} + \beta \tau Z = \beta R + \rho \beta \tau Z + \beta \tau Z. \]

Solving this for \( \tau \) with \( R = L \), we obtain the price of permits with entry/closure provision given the emissions cap \( Z \):

\[ \tau_{OBA} = \frac{\beta L}{1 - \beta(1 + \rho)} \] \[ (A2) \]

Comparing this with (17), we see \( \tau_{OBA} > \tau_A \). Hence, the price of permits is higher under the OBA rule than under the auctioned ET.

Furthermore, following the same steps as for \( M_A \), we have:

\[ M_{OBA} = \frac{(1 - \beta(1 + \rho - \sigma))L}{\sigma(1 - \beta(1 + \rho))(\pi_{OBA} + f \tau^\beta)} = \frac{(1 - \beta(1 + \rho - \sigma))L}{\sigma(1 - \beta(1 + \rho))(\phi_{OBA})^{\sigma - 1}} \left( f \tau^\beta \right). \] \[ (A3) \]

Comparing this with (18), we observe that

\[ M_A / M_{OBA} = \frac{1}{1 - \beta(1 + \rho - \sigma)} \left( 1 - \beta(1 + \rho) \right)^{1 - \beta} \left( 1 - \frac{1 - \beta(1 + \rho)}{1 - \beta} \right) < 1, \]

where the last inequality follows because both the first and second terms are less than 1 (for the first term, note \( 1 - \beta(1 + \rho - \sigma) = 1 + \beta \rho^2 \sigma > 1 \)).
A.3. Proof of Proposition 5

To show \( Q_{CP} \leq Q_A \leq Q_{OBA} \) given \( Z \): Consider \( Q_{CP} / Q_A \). Substitute \( R_A = L/(1 - \beta) \) and \( R_{CP} = L \) and mass-of-firms expressions (18) and (A1) into (34) while the last three terms equal 1. Then

\[
Q_{CP} / Q_A = (1 - \beta)(1 - \beta + \sigma \beta)^{1/(\sigma - 1)}.
\]

To see this expression \( \leq 1 \), note that it equals 1 when \( \beta = 0 \) and its derivative is negative:

\[
\frac{\partial Q_{CP}}{\partial \beta} = \left[ 1 + (\sigma - 1)\beta \right]^{1/(\sigma - 1)} \left\{ -1 + (1 - \beta)[1 + (\sigma - 1)\beta]^{-1} \right\},
\]

where the first multiplicative term is positive and the second multiplicative term is

\[-1 + (1 - \beta)[1 + (\sigma - 1)\beta]^{-1} \leq 0.\]

Now consider \( Q_{OBA} / Q_A \). Substitute \( R_A = L/(1 - \beta) \) and \( R_{OBA} = L \), mass-of-firms expressions (18) and (A3), and permit-price expressions (17) and (A2) into (34). Manipulating the terms, we obtain

\[
Q_{OBA} / Q_A = (1 - \rho \beta) \left( \frac{1 - \beta(1 + \rho)}{1 - \beta} \right)^{(\beta - 1)/\rho} \left( \frac{1 - \rho + \rho^2 \beta}{1 - \rho} \right)^{1/(\sigma - 1)}.
\]

To see this \( \geq 1 \), first note that for any \( \rho < 1 \), it equals 1 when \( \beta = 0 \) (Note: we have \( \tau_{OBA} > 0 \) when \( \beta < 1/(1 + \rho) \), which we assume to hold in this paper). Hence, this expression is always greater than or equal to 1 if its derivative is positive. To show the derivative is indeed positive, let

\[
A \equiv \left( \frac{1 - \beta(1 + \rho)}{1 - \beta} \right)^{(\beta - 1)/\rho},
\]

\[
B \equiv 1 - \rho \beta, \quad \text{and}
\]

\[
C \equiv \left( \frac{1 - \rho + \rho^2 \beta}{1 - \rho} \right)^{1/(\sigma - 1)}.
\]

Then \( Q_{OBA} / Q_A = ABC \) and we have

\[
\frac{\partial Q_{OBA}}{\partial \beta} = A'B'C + AB'C + ABC' = ABC \left\{ \frac{1}{\rho} \log \left( \frac{1 - \beta(1 + \rho)}{1 - \beta} \right) + \frac{1}{1 - \beta(1 + \rho)} \right\}
\]

\[-\rho AC + ABC \left( \frac{\rho(1 - \rho)}{1 - \rho + \rho^2 \beta} \right),
\]

where in the first term, we applied a rule of differentiation:
\[
\frac{d}{dx}(A(x))^{f(x)} = (A(x))^{f(x)}
\]

\[
\left[f'(x)\log A(x) + f(x)\frac{A'(x)}{A(x)}\right].
\]

Because \(1 - 1/x < \log(x)\) for \(0 < x < 1\), the first term inside the braces satisfies

\[
\frac{1}{\rho} \log \left(\frac{1 - \beta(1 + \rho)}{1 - \beta} \right) > \frac{1}{\rho} \left(1 - \frac{1 - \beta}{1 - \beta(1 + \rho)} \right) = \frac{-\beta}{1 - \beta(1 + \rho)}.
\]

Hence,

\[
\frac{\partial Q_{OBA}/Q_A}{\partial \beta} > ABC \left(\frac{1 - \beta}{1 - \beta(1 + \rho)} \right) - ABC \left(\frac{\rho(1 - \beta)}{1 - \beta + \rho^2 \beta} \right)
\]

\[
= \frac{Q_{OBA}}{Q_A} \left(\frac{1 - \beta}{1 - \beta(1 + \rho)} \right) - \rho^2 \beta \left(1 - \beta(1 + \rho) \right) \left(1 - \rho^2 \beta \right) > 0.
\]

Therefore, \(Q_A > Q_{OBA}\) holds given any combinations of \(\beta\) and \(\rho\) (such that \(\tau_{OBA} > 0\)).

To show the second-best optimum satisfies \(Z_{CP} \leq Z_A \leq Z_{OBA}\): Provided that the disutility from pollution \(h(\cdot)\) be given by \(h(Z) = Z^a (a \geq 1)\), the first-order necessary condition for the program (35) for scheme \(i\) is

\[
(\beta/\rho) \nu_i \Gamma(\phi_i^{(\beta/\rho)}) = a Z_i^{\beta-1}.
\]

This first-order condition implies:

\[
Z_{CP}/Z_A = (\nu_{CP})^{\rho/(\mu - \beta)} \quad \text{and} \quad Z_{OBA}/Z_A = (\nu_{OBA})^{\rho/(\mu - \beta)}.
\]

Because \(\nu_{CP}\) is less than 1 and \(\nu_{OBA}\) is greater than 1 as shown above, the second-best optimum satisfies \(Z_{CP} \leq Z_A \leq Z_{OBA}\) if \(a \geq \beta/\rho\). Note that the second-order condition is

\[
(\beta/\rho) (\beta/\rho - 1) \Gamma(\phi_i^{(\beta/\rho)}) Z_i^{\beta(\beta/\rho) - 2} - a(a - 1) Z_i^{\beta - 2} \leq 0.
\]

Plugging in the first-order condition and manipulating, we have...
which requires \( a \geq \beta / \rho \).

**A4. Proof of Proposition 6**

Consider first the economy under auctioning. Aggregating \( \tau Z^k \) for each sector, we see \( \tau Z^k = \rho \beta^k R^k \). The Cobb-Douglas specification for fixed costs of production and entry implies that \( \tau Z^k = \beta^k M^k \sigma^k \) and \( \tau Z^k = \beta^k R^k \). Using the fact that \( \pi^k = [(\phi^k / \phi^m)^{\sigma^k - 1} - 1] \tau \sigma^k \) and manipulating, we obtain \( \tau Z^k = \beta^k R^k \). Summing this over the two sectors, and applying \( R^k = \alpha^k R \),

\[
\tau Z = \theta R = \theta (L + \tau Z).
\]

Solving this for \( \tau \), we obtain the equilibrium price of permits as in the main text:

\[
\tau^A = \frac{\theta L}{(1 - \theta)Z}.
\]

Furthermore, rearranging this, we see \( L = (1 - \theta)\tau Z / \theta \). Substituting this into \( \tau Z^k = \alpha^k \beta^k R^k = \alpha^k \beta^k (L + \tau Z) \), we obtain the equilibrium emissions in each sector:

\[
Z^k_A = \frac{\alpha^k \beta^k}{\theta} Z.
\]

Similarly, consider the accounting equation for labor. Applying analogous logics, we have \( L^k = (1 - \beta^k)R^k = \alpha^k (1 - \beta^k) R \). Substituting \( \tau Z = \theta L / (1 - \theta) \), we obtain the equilibrium labor employment in each sector:

\[
L^k_A = \frac{\alpha^k (1 - \beta^k)}{1 - \theta} L.
\]

This also implies that \( R^k = \alpha^k L / (1 - \theta) \). Hence, applying this in \( M^k = R^k / \tau^k \), we see:

\[
M^k_A = \frac{\alpha^k L}{\sigma (1 - \theta)(\pi^k_A + \tau^\pi^k_A)}.
\]

Next, consider the economy under grandfathering with closure provision. Applying analogous logics, the accounting equation for aggregate emissions can be written as:

\[
\tau Z^k = \rho \beta^k R^k + \beta^k M^k \sigma^k \tau + \beta^k M^k \tau^\pi^k_A,
\]

\[
= \rho \beta^k R^k + \beta^k M^k (\tau \sigma^k A^k + \tau Z^k),
\]

\[
= \rho \beta^k R^k + \beta^k (R^k / \sigma + \tau A^k Z),
\]

\[
= \beta^k R^k + \tau A^k \beta^k Z.
\]
Summing this over sectors and applying $R^k = \alpha^k R$,

$$\tau Z = \theta R + \tau \theta Z = \theta L + \tau \theta Z.$$ 

Solving this for $\tau$, we obtain the equilibrium price of permits as in the main text:

$$\tau_{CP} = \frac{\theta L}{(1 - \theta)Z}.$$ 

Furthermore, substituting this into $\tau Z^k = \beta^k R^k + \tau \alpha^k \beta^k Z = \alpha^k \beta^k L + \tau \alpha^k \beta^k Z$, we obtain the equilibrium emissions in each sector:

$$Z^k_{CP} = \frac{\alpha^k \beta^k (1 - \theta) + \alpha^k \beta^k \theta}{\theta}Z.$$ 

Similarly, applying analogous logics, we have $L^k = \alpha^k (1 - \beta^k) R + \tau \alpha^k (1 - \beta^k) Z$. Substituting $\tau Z = \theta L / (1 - \theta)$, we obtain the equilibrium labor employment in each sector:

$$L^k_{CP} = \frac{[\alpha^k (1 - \beta^k) (1 - \hat{\theta}) + \alpha^k (1 - \beta^k) \theta] L}{1 - \theta}.$$ 

Last, to solve for the equilibrium mass of firms, note that $R^k = \alpha^k R = \alpha^k L$. Hence, applying this in $M^k = R^k / \tau^k$, the equilibrium mass of firms is

$$M^k_{CP} = \frac{\alpha^k L}{\sigma(\pi^k_{CP} + f^{\tau^k_{CP}})}.$$ 

Substituting $\tau^k = \hat{\alpha}^k Z / M^k$ and solving for $M^k_{CP}$, we obtain

$$M^k_{CP} = \frac{\alpha^k (1 - \hat{\theta}) + \sigma \hat{\alpha}^k \theta}{\sigma (1 - \theta)(\pi^k_{CP} + f^{\tau^k_{CP}})}.$$ 

Applying $\pi_i = [(\phi_i / \phi^*_i)^{\mu - 1} - 1] f \tau^\mu_i$ with $\phi^*_i = \phi^*_i$ and the expressions for $\tau_A$ and $\tau_{CP}$, we have

$$M^k_A / M^k_{CP} = \frac{\alpha^k}{\alpha^k (1 - \hat{\theta}) + \sigma \hat{\alpha}^k \theta} \left( \frac{1 - \hat{\theta}}{1 - \theta} \right)^{1 - \beta^k}.$$ 

This equation implies, first, that $M^k_A < M^k_{CP}$ if $\hat{\alpha}^k = \alpha^k$ as expected from the one-sector model and, second, that whether $M^k_{CP}$ exceeds $M^k_A$ is, in general, indeterminate and depends on the size of $\hat{\alpha}^k$. 
REFERENCES


Impact of Permanent Allocation Rule

In this online appendix, we outline the proof that the stationary equilibrium outcome under emissions trading is the same between auction and permanent permit allocation. With permanent allocation, firms that receive permits retain the permits upon exit, whereas active firms that do not receive permits need to buy permits every period from other firms that hold them. In other words, the initial distribution of permits is permanent regardless of firms’ entry/exit status. Such a rule is used in the US Acid Rain Program.

A point of departure for our analysis is the entry-exit condition embodied in (ZCP) and (FE). Because the firms that receive permits upon entry can sell permits upon exit, the exit condition for such firms under the permanent allocation (PA) is

\[ \frac{r(\phi^*)}{\sigma} - f \tau^\beta + \tau z^*(\phi^*) = \tau z^*(\phi^*_p). \]

On the other hand, the firms that do not receive permits upon entry must buy permits in the auction or emissions market. Hence, the exit condition for such firms is identical to the case of auctioning:

\[ \frac{r(\phi^*)}{\sigma} - f \tau^\beta = 0. \]

Either way, firm’s economic profit is \( \pi(\phi) = r(\phi)/\sigma - f \tau^\beta \). Consequently, the zero-cutoff productivity is identified by \( r(\phi^*) = \sigma f \tau^\beta \), which is identical to that in the case of auctioning. Following the same steps as under the auctioned ET, we obtain the same entry-exit equation as that of the auctioned ET. Hence, the cutoff productivity stays the same as under the auctioned ET.

How about the mass of firms and the price of permits? Consider the aggregate profit:

\[ \Pi = \int \pi(\phi) M \mu(\phi) d\phi = M \pi. \]

In this expression, there are firms that receive permits for free as well as firms that do not. Either way, their economic profits must equal \( \pi(\phi) = r(\phi)/\sigma - f \tau^\beta \). Hence, the aggregate profit must equal the aggregate revenues from sales of commodities and permits less the sum of payments to labor and emissions used in production and the aggregate opportunity cost of staying active in the industry: \( \Pi = R + \tau \bar{Z} - L_p - \tau Z_p - \tau \bar{Z} \), where \( \bar{Z} \) is the total amount of permits that were freely distributed to the firms that stay active.\(^{14}\) Equivalently, \( L_p + \tau Z_p = R - \Pi \). Following the same logics as in the case of auctioning, we obtain the same expression for aggregate income as under auctioning. As for the economy-wide demand for permits, we obtain the same expression as in the case of auctioning because the average (economic) profit \( \bar{\pi} = (\pi/\sigma) - f \tau^\beta \) remains the same. Therefore, both the mass of firms and the price of permits are the same as those under the auctioned ET.

Importantly, this stationary equilibrium does not depend on the mass of active firms that receive permits freely. The key here is that all active firms’ economic profits under the permanent allocation rule can be written the same way as those under auctioning regardless of whether they receive permits freely or not. Hence, their economic behavior is completely unaffected. Moreover, because these arguments do not depend on how permits are distributed initially to which firms, the independence property holds under the permanent allocation rule.

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\(^{14}\) Note that this expression does not depend on how many of these firms who receive permits for free stay active in equilibrium. If there are no such active firms, \( Z = 0 \) and the value of permits held by inactive firms would be simply accounted for in the demand side.
Appendix B from Konishi and Tarui “Emissions Trading, Firm Heterogeneity, and Intra-industry Reallocations in the Long Run”

**Proposition B1:** Given the same cap of aggregate emissions $Z$, the stationary equilibrium outcome under emissions trading is the same between auction and permanent permit allocation (i.e., $\phi_A = \phi_P^*$, $M_A = M_P$, $N_A = N_P$, and $\tau_A = \tau P A$) regardless of the initial distribution of permits.

**Results with the CES Production Function**

Next, we demonstrate that the main results of our paper still hold when the production function is CES. Suppose the production function of firm with productivity $\phi$ is given by:

$$q = \begin{cases} 
\phi \{ \beta z \rho_p + (1 - \beta) l^\rho \}^{1/\rho} & \text{if } z < \lambda l \\
\phi A l & \text{otherwise}
\end{cases},$$

where $A \equiv \{ \beta \lambda^\rho + (1 - \beta) \}^{1/\rho}$ and $\rho_p < 1$. The elasticity of input substitution is given by $\sigma_p \equiv 1/(1 - \rho_p)$. We focus on the case where $z < \lambda l$ in equilibrium. From the first-order conditions of the cost minimization, we have

$$z \Gamma = \left( \frac{w}{\lambda - \beta} \right)^{\sigma},$$

$$q = \phi \{ \beta z \rho_p + (1 - \beta) l^\rho \}^{1/\rho}.$$

With manipulation, we obtain the marginal cost of production $c(\tau, w)/\phi$ where

$$c(\tau, w) = \{ \beta^\sigma \lambda^{1 - \sigma} + (1 - \beta)^{\sigma} l^{1 - \sigma} \}^{1/(1 - \sigma)}.$$

The markup pricing rule stays the same as in the case of a Cobb-Douglas production function except that $\tau^\beta l^{1 - \beta}$ is now replaced by $c(\tau, w)$:

$$q(\phi) = Q \left( \frac{P \rho \phi}{c(\tau, w)} \right)^{\sigma}, \quad p(\phi) = \frac{c(\tau, w)}{\rho \phi}, \quad r(\phi) = R \left( \frac{P \rho \phi}{c(\tau, w)} \right)^{\sigma - 1},$$

where $Q$, $P$, $R$, $\rho$, and $\sigma$ are as defined in the main text.

Now we examine whether the equilibrium differs qualitatively under the alternative assumption of the CES technology. First, we compute the equilibrium cutoff productivity level under auctioned emissions trading. The above expressions indicate that given any two productivity levels $\phi$ and $\phi'$, the ratios of outputs $q(\phi')/q(\phi)$, revenues $r(\phi')/r(\phi)$, and variable inputs depend on the ratio of the productivity levels $\phi'/\phi$ and the elasticity of consumption substitution $\sigma$—the same property as in the Cobb-Douglas case. We also observe that these ratios are independent of the elasticity of input substitution $\sigma_p$ and the factor share parameter $\beta$. This implies that the equilibrium cutoff productivity will be the same as in the case of the Cobb-Douglas technology. Indeed, the ZCP condition:

$$\Pi = \pi(\phi) = \left[ \left( \frac{\phi}{\phi'} \right)^{\sigma - 1} - 1 \right] f(\tau, w),$$

and the FE condition:

$$(1 - G(\phi^*)) \left( \frac{\phi}{\phi^*} \right)^{-1} f_c(\tau, w) = 0,$$

now have $c(\tau, w)$ instead of $\tau^\beta$. However, the cutoff productivity is the same as in the case of the Cobb-Douglas technology:

$$(1 - G(\phi^*)) \left[ \left( \frac{\phi}{\phi^*} \right)^{-1} - 1 \right] = \frac{\delta \phi}{\gamma}.$$
The same applies to the determination of $\phi^*$ under other allocation rules. Thus allowing technology to be CES does not influence the determination of $\phi^*$.

Next, we compute the equilibrium price of permits under each emissions trading scheme. The above cost-minimization solution implies that the factor payment on emissions for the variable part of production is given by:

$$\tau_{\phi v}(\phi) = h(\tau, w)q(\phi)/\phi = \rho h(\phi),$$

where $h$ is the factor payment share defined by

$$h = \frac{1}{1 + \left(\frac{\tau}{w}\right)^{\rho/(1-\rho)} \left(\frac{1-\beta}{\beta}\right)^{1/(1-\rho)}}.$$

Analogously, the factor payment on emissions for the fixed cost of production is

$$\tau_{\phi f} = h(\tau, w).$$

(As in the main text, we assume that the emission intensity is the same for production and entry.) Then the cost share of emissions in the entry cost is given by

$$\tau_e = h\pi_\text{e}. $$

Aggregate these values over firms to obtain

$$\tau Z = \tau Z_{\phi v} + \tau Z_{\phi f} + \tau Z_e,$$

$$\tau Z = \rho h R + h M \tau c(\tau, w) + h M \pi,$$

$$\tau Z = \rho h R + h R \tau c(\tau, w) + h R \pi,$$

where $\tau = \sigma(\pi + fc(\tau, w))$. Hence

$$\tau Z = \rho h R + \frac{h R \tau c(\tau, w) + h R \pi}{\sigma(\pi + fc(\tau, w))} = \rho h R + (1-\rho)h R = h R,$$

where $R = \tau Z + w L$. Plug in the definition of $h$ to the expression and we have

$$\tau Z \left[ 1 + \left(\frac{\tau}{w}\right)^{\rho/(1-\rho)} \left(\frac{1-\beta}{\beta}\right)^{1/(1-\rho)} \right] = \tau Z + w L.$$

By letting $w = 1$ and solving for $\tau$, the equilibrium permit price under auction is given by

$$\tau_A = \frac{\beta}{1-\beta} \left(\frac{1}{2}\right)^{1-\rho}.$$

Note that this expression is identical to the Cobb-Douglas case when $\rho \rightarrow 0$. The above derivation indicates that propositions 1 and 2 would remain intact under the assumption of the CES technology.

Under the closure provision, we have

$$\tau = \sigma(\pi + fc(\tau, w) - \tau_{\phi v}),$$

$$= \sigma\left(\pi + fc(\tau, w) - \tau Z M \right),$$

$$= \sigma(\pi + fc(\tau, w)) - \tau_{\phi v} Z_{\phi v}.$$
and hence
\[ r = \frac{\sigma(\tau + fc(\tau, w))}{1 + \sigma Z R}. \]

Apply this in
\[ \tau Z = \rho R + \frac{h R}{\frac{\tau}{\sigma} + fc(\tau, w)} + \frac{h R}{\frac{\tau}{\sigma} \pi}, \]
\[ = \rho R + \frac{h R fc(\tau, w) + h R \pi}{\sigma(\tau + fc(\tau, w))}, \]
\[ = \rho R + \frac{h R}{\sigma} \left( 1 + \frac{\sigma Z}{R} \right), \]
\[ = \rho R + (1 - \rho) h R + h \tau Z. \]

Hence, the demand for permits is given by
\[ \tau Z = \frac{h}{1 - h} R \left( \frac{\tau}{\rho} \right)^{1/(1-\rho)} \frac{1}{\left( \frac{1 - \beta}{\beta} \right)^{1/(1-\rho)}} R. \]

Because \( R = wL \) under closure provision of permits, we have (again with \( w = 1 \)):
\[ \tau_{CP} = \frac{h}{1 - h} \frac{L}{Z}. \] (B1)

Using the definition of \( h \) and manipulating, we obtain
\[ \tau_{CP} = \frac{\beta}{1 - \beta} \left( \frac{L}{Z} \right)^{-\rho} = \tau_4. \]

It follows that proposition 3 remains intact: the equilibrium permit price is the same under auction and under closure provision.

Last, under the output-based allocation, we have
\[ \tau_{Zpv} = \rho h \gamma R, \]
where \( \gamma \equiv 1 + (\tau Z)/R. \) Therefore,
\[ \tau Z = \rho h \left( 1 + \frac{\tau Z}{R} \right) R + (1 - \rho) h R + h \tau Z = h R + \rho h \tau Z + h \tau Z, \]
and hence
\[ \tau_{OBA} = \frac{h}{1 - h(1 + \rho)} \frac{L}{Z}. \] (B2)

Comparing (B1) and (B2), we see that the result that \( \tau_{OBA} > \tau_4 = \tau_{CP} \) continues to hold under the assumption of the CES technology.